Calibrate Time Series by the LSTM Model

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Introduction

- In Econometrics, there are various time series models and the theory of estimation guarantees is tailored to each one
- Herein we study calibration of simulated time series by the generic LSTM model. Further more, we simulate time series with latent states and check how well we learnt the latent states.

Methodology

- Peepholed-version of LSTM

$$
\begin{aligned}
f_t &= \sigma_g(W_f x_t + U_f c_{t-1} + b_f) \\
i_t &= \sigma_g(W_i x_t + U_i c_{t-1} + b_i) \\
o_t &= \sigma_g(W_o x_t + U_o c_{t-1} + b_o) \\
c_t &= f_t \circ c_{t-1} + i_t \circ \sigma_c(W_c x_t + b_c) \\
h_t &= o_t \circ \sigma_h(c_t)
\end{aligned}
$$

- Simulate 5000 trajectories for training, 500 for test, each with 1000 time steps

$$y_t = f\left(y_{t-p}, \cdots, y_{t-1}, \epsilon_{t-q}, \cdots, \epsilon_{t-1}\right) + \epsilon_t$$

$$y_{t|t-1} = f\left(y_{t-p}, \cdots, y_{t-1}, \epsilon_{t-q}, \cdots, \epsilon_{t-1}\right)$$

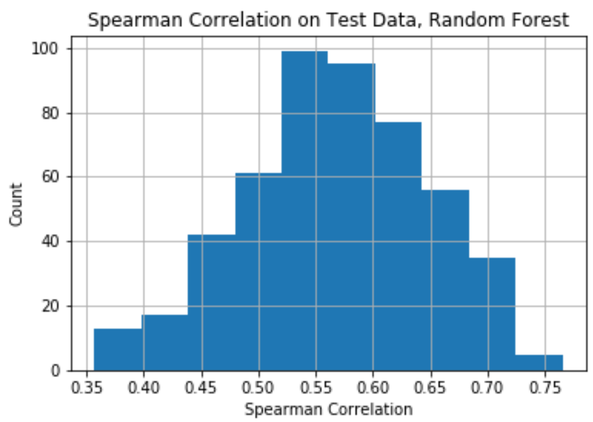
- Loss metrics: RMSE & 50%-Quantile
- SGD with Momentum, Momentum param = 0.5
- The first 50 time steps on any trajectory are for “burn-in”

An ARIMA(2,0,2) Process

- ARIMA(2,0,2) with params that mimic S&P 500 daily return series since 1980
- Symmetric and asymmetric innovation distributions
- After fitting LSTM, the error w.r.t. $y_{t|t-1}$ is centred (for sym. innov. distribution) and orthogonal to the ϵ_t used for simulation
- This motivates Model Averaging. It usually takes 20 runs to produce stable fitting error w.r.t. $y_{t|t-1}$

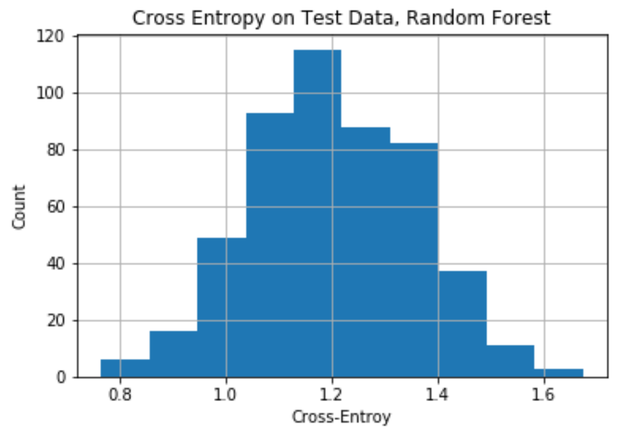
A Stochastic Volatility Process

- AR(2) with innovation terms following mean-reverting variance
- Explain the ground-truth latent variance by Linear Regression and Random Forecast Regressor
- Spearman rank-based correlation on the Test traiectories: 0.55 +/- 0.2



A Regime-Switching Process

- AR(2) with innovation terms following 2-regime switching process
- Classify the groud-truth regime with fitted LSTM by Logistic Classifier and Random Forest Classifier
- Cross-Entropy Logistic: 0.64+/-0.1 RF: 1.2+/-0.4
- Accuracy Logistic: 0.65+/-0.1, RF: 0.72+/-0.08
- Accuracy at the expense of Cross-Entropy



Conclusion

- Model Averaging helps when training LSTM on time series
- Weak to moderate evidence that LSTM completely “understood” and learnt the internal structure of simulated time series