Decision Trees

CS229: Machine Learning

Carlos Guestrin
Stanford University

Slides include content developed by and co-developed with Emily Fox

Predicting potential loan defaults

What makes a loan risky?

- Credit History ★★★★
- Income ★★★
- Term ★★★★★
- Personal Info ★★★

Credit history explained

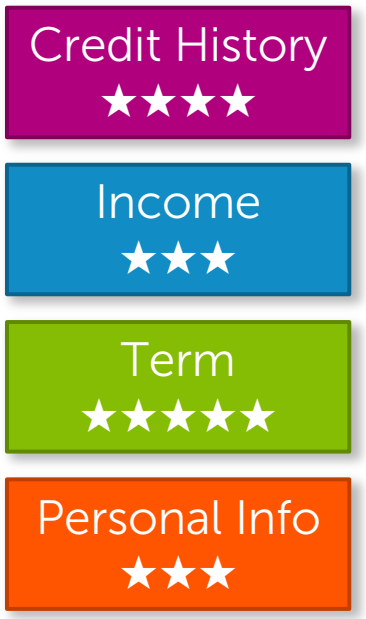
Did I pay previous loans on time?

Example:
excellent, good, or fair

Income

What's my income?

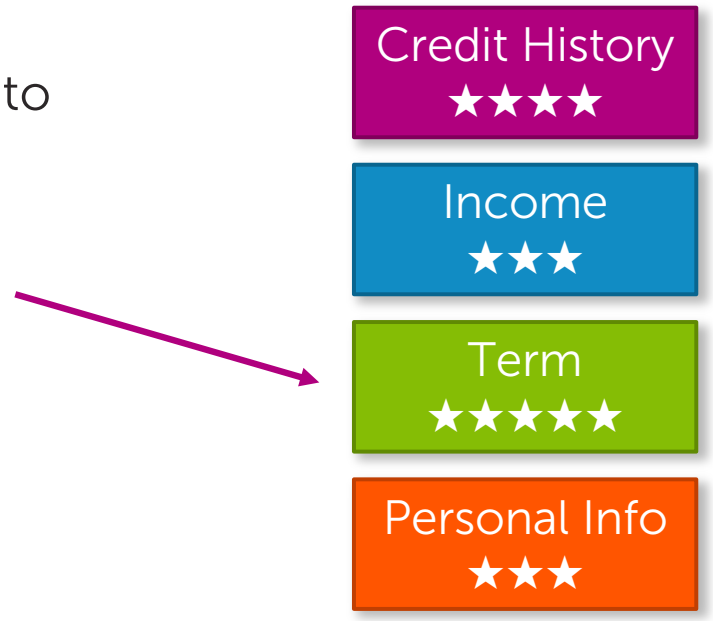
Example:
$80K per year



Loan terms

How soon do I need to pay the loan?

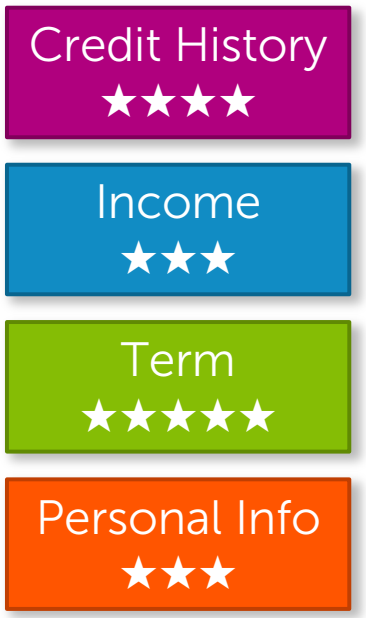
Example: 3 years, 5 years,...



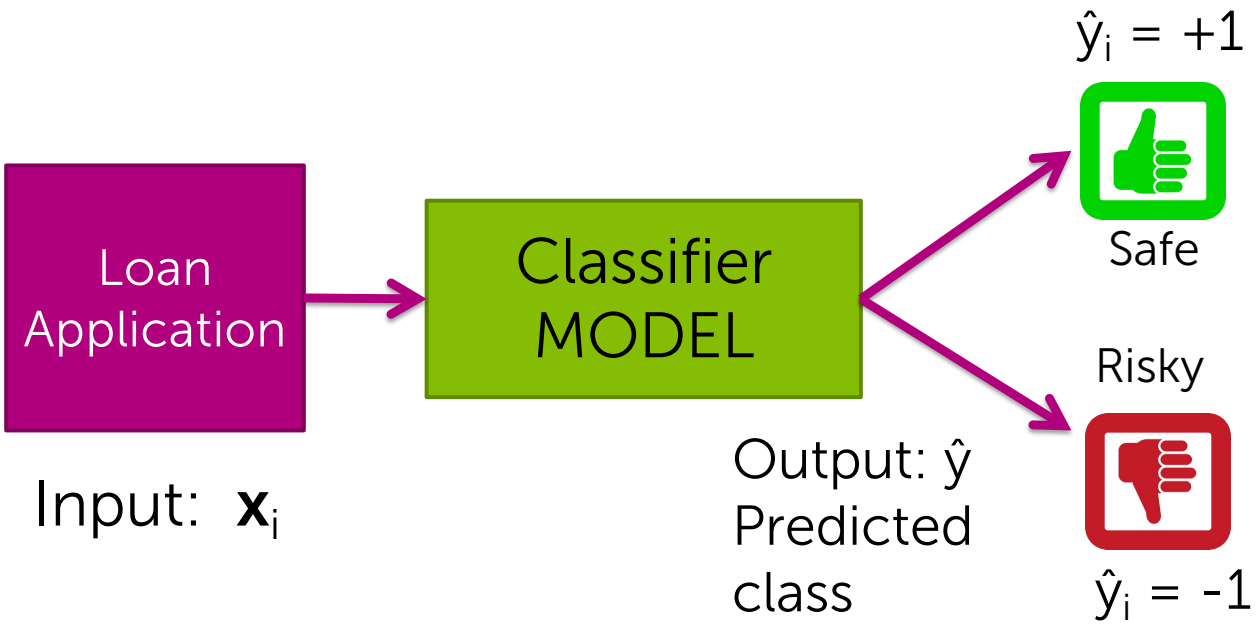
Personal information

Age, reason for the loan, marital status,…

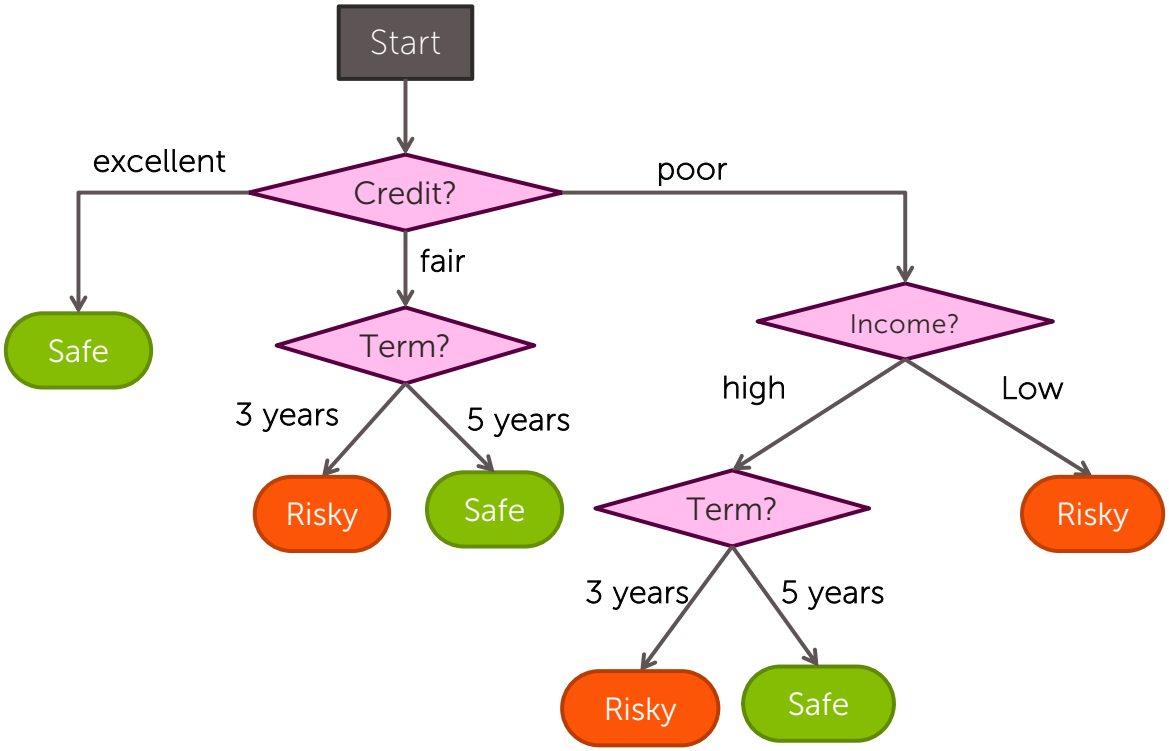
Example: Home loan for a married couple



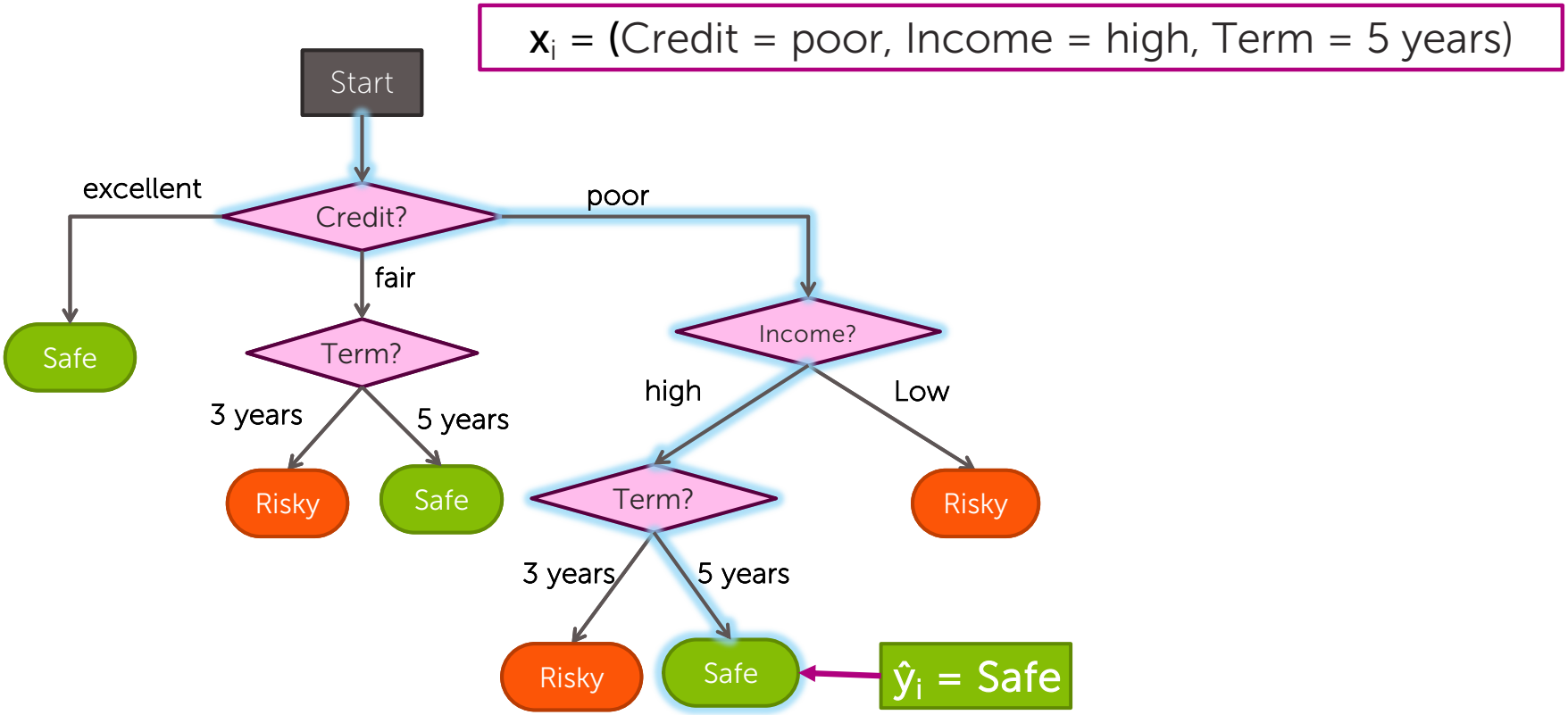
Classifier review



This module … decision trees



Scoring a loan application

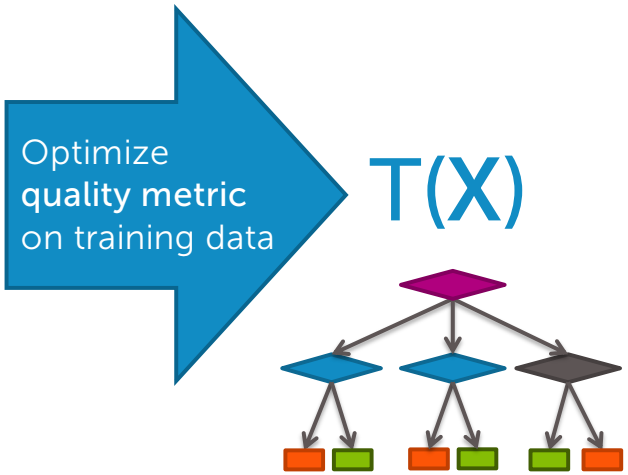


Decision tree learning task

Decision tree learning problem

Training data: N observations ($\mathbf{x}_i, y_i$)

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe



Quality metric: Classification error

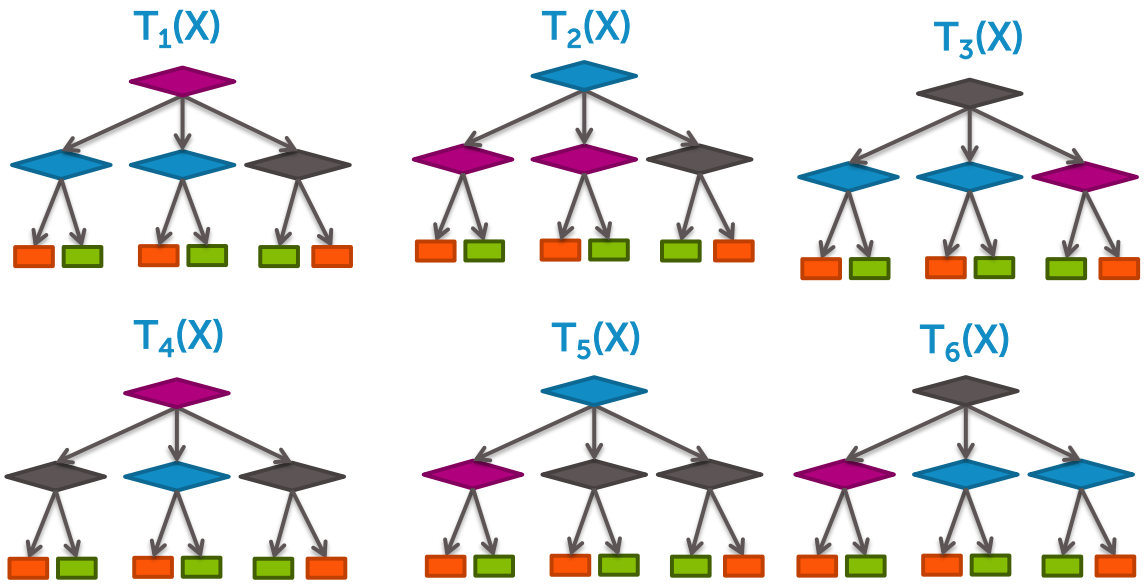
- Error measures fraction of mistakes

$$\text{Error} = \frac{\text{\# incorrect predictions}}{\text{\# examples}}$$

  - Best possible value : 0.0
  - Worst possible value: 1.0

How do we find the best tree?

Exponentially large number of possible trees makes decision tree learning hard!



Learning the smallest decision tree is an *NP-hard problem* [Hyafil & Rivest '76]

Greedy decision tree learning

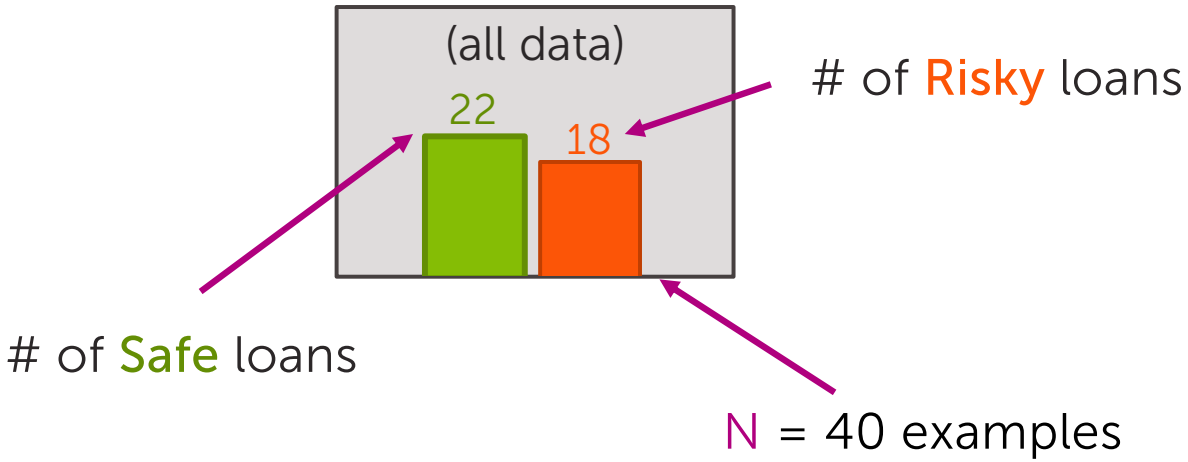
Our training data table

Assume N = 40, 3 features

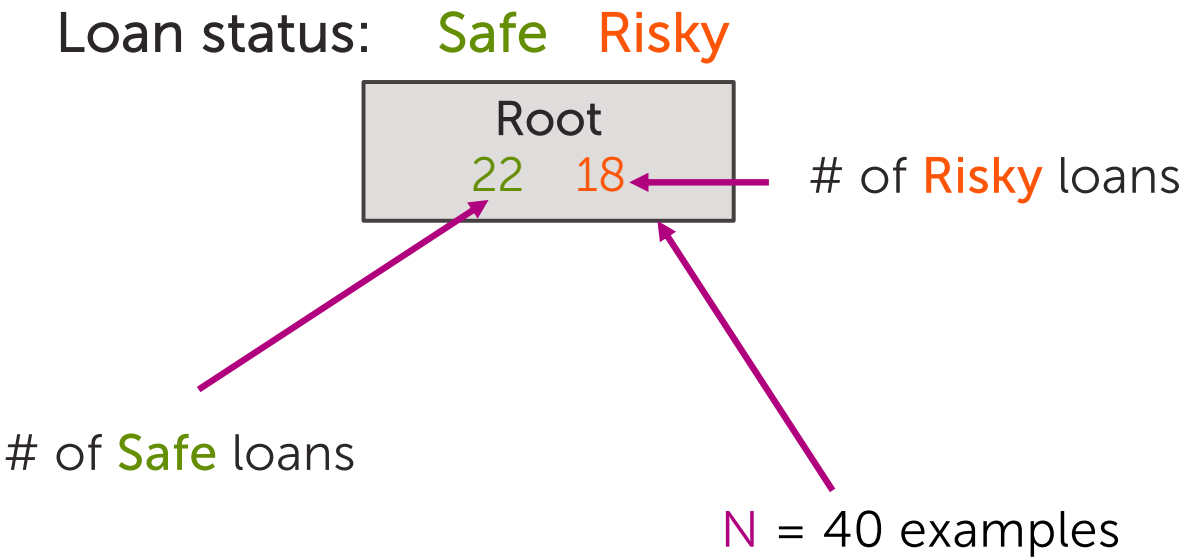
Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe

Start with all the data

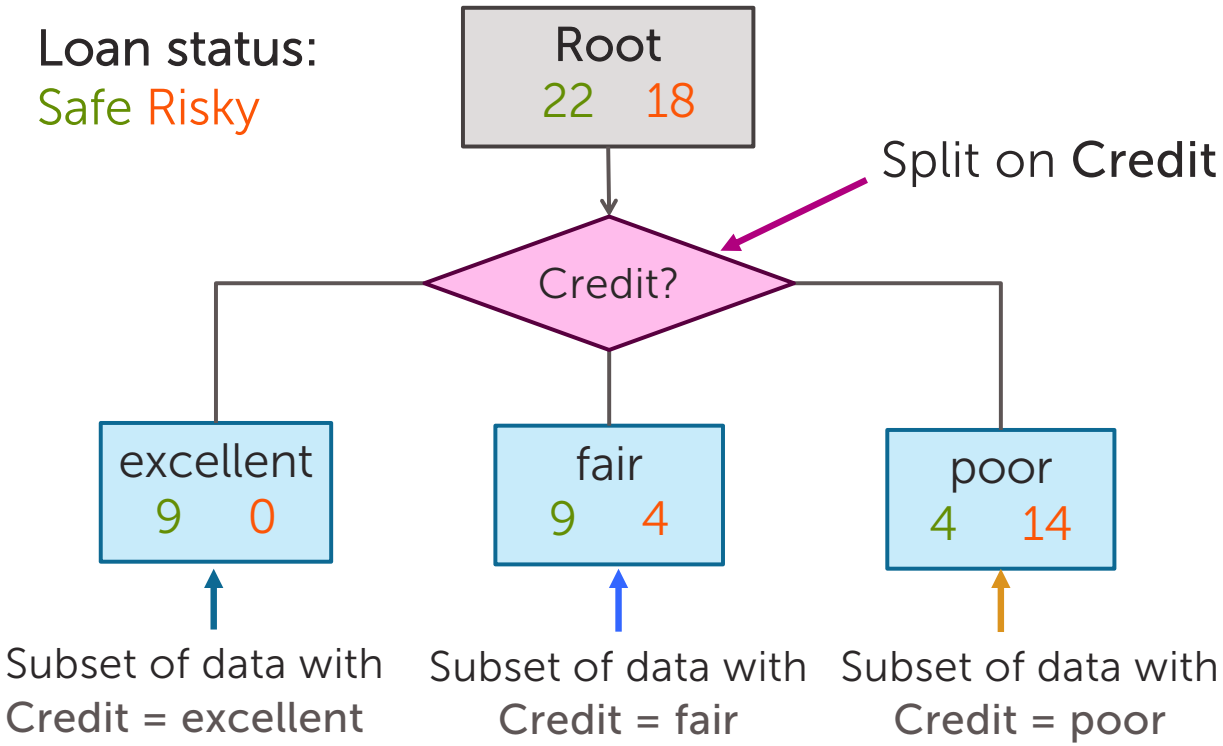
Loan status: Safe Risky



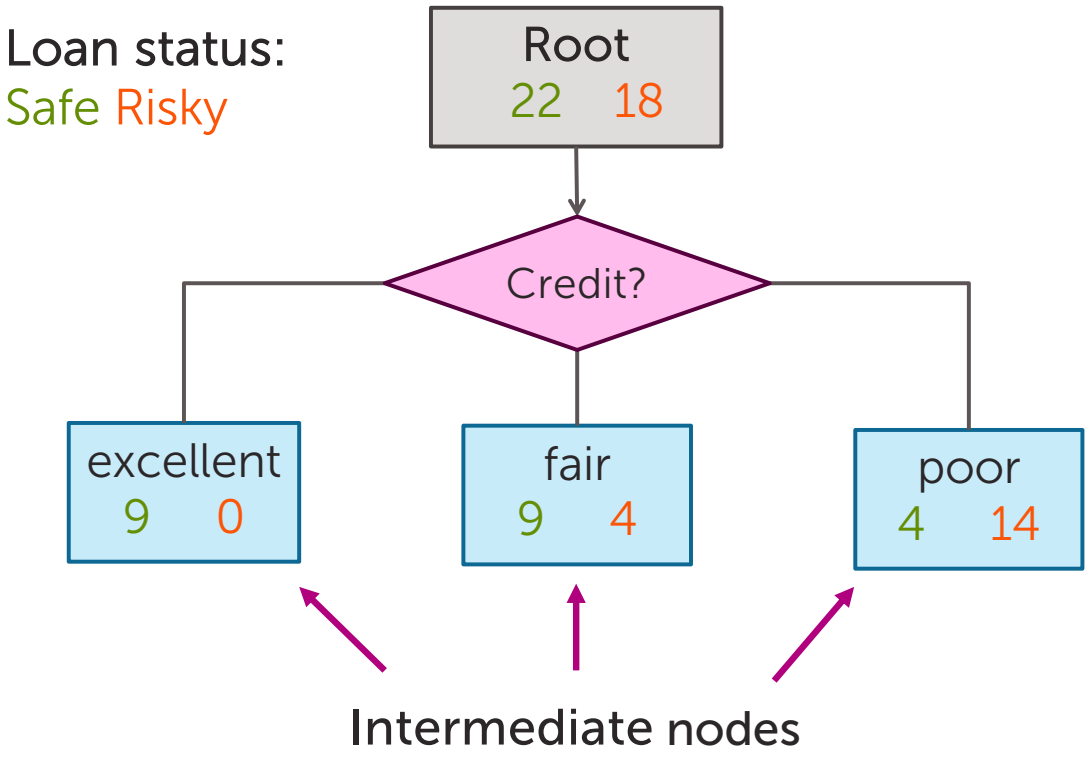
Compact visual notation: Root node



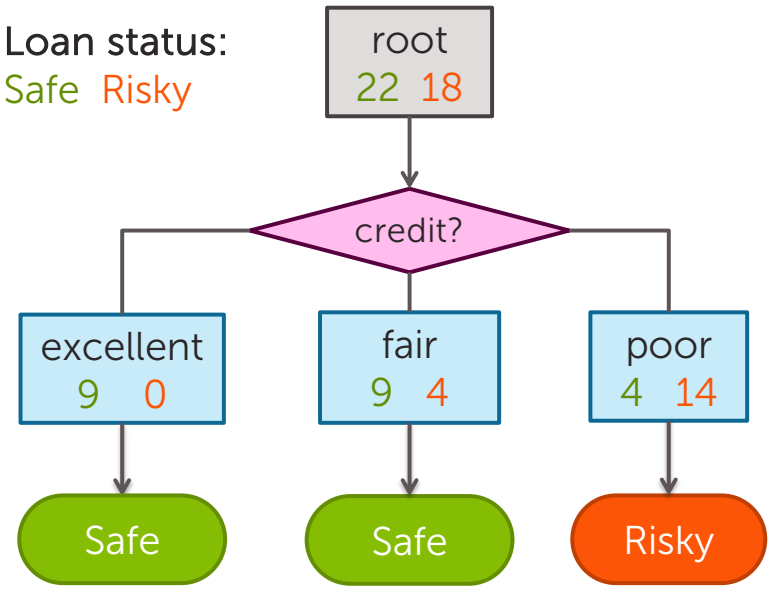
Decision stump: Single level tree



Visual notation: Intermediate nodes



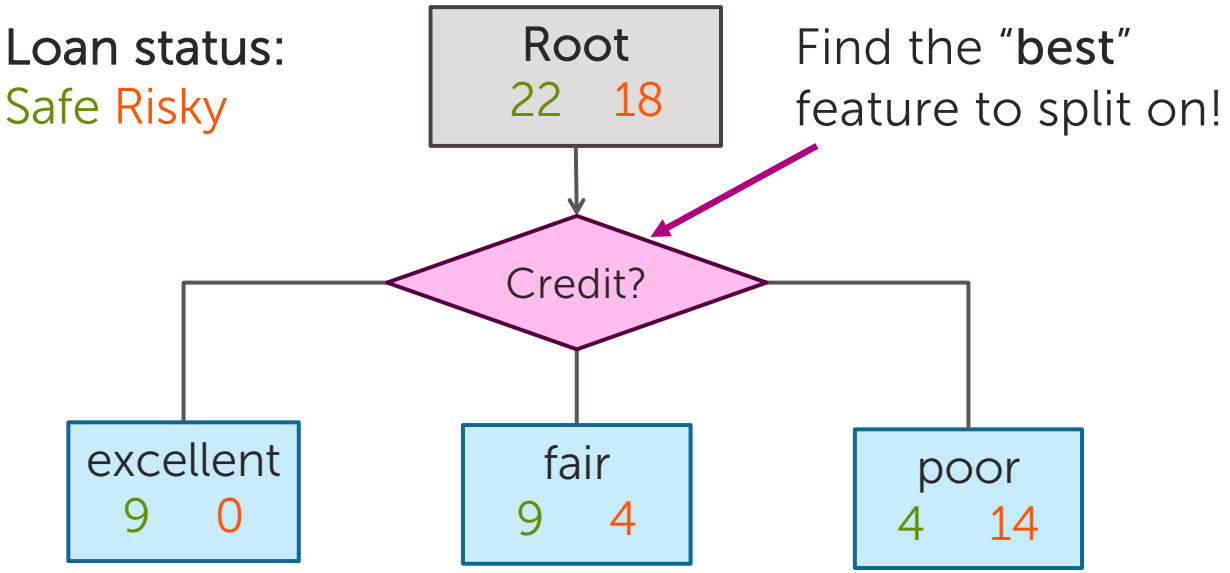
Making predictions with a decision stump



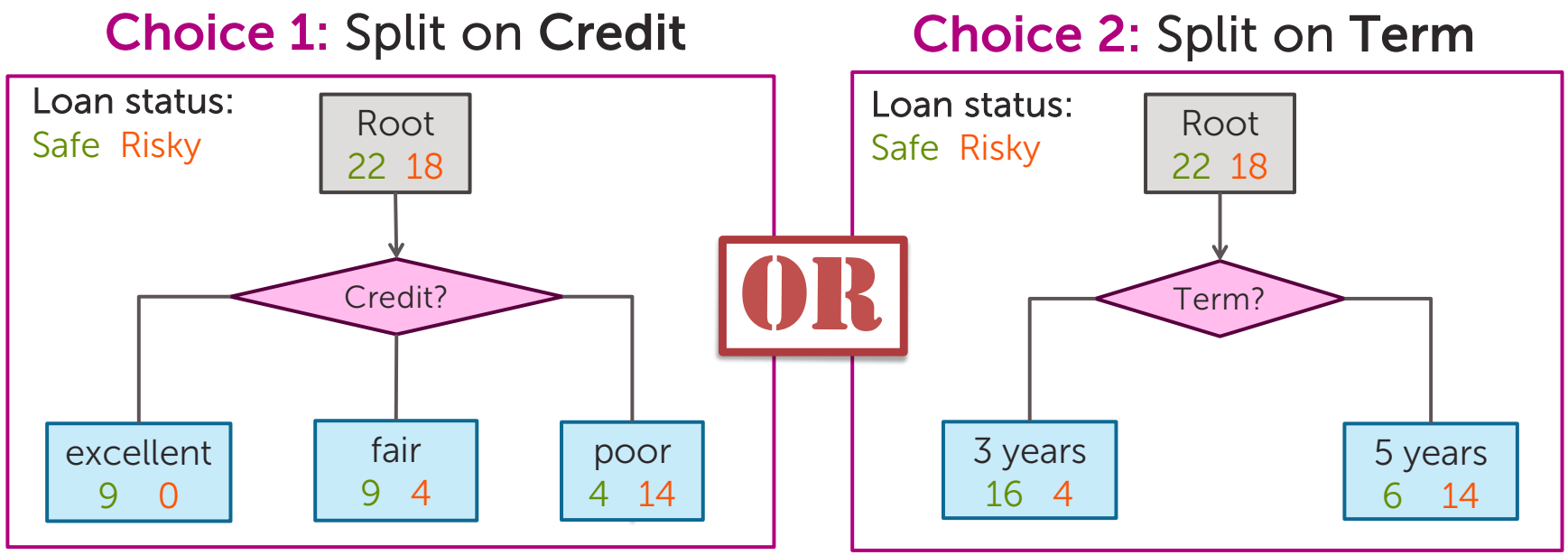
For each intermediate node,
set $\hat{y}$ = **majority value**

Selecting best feature to split on

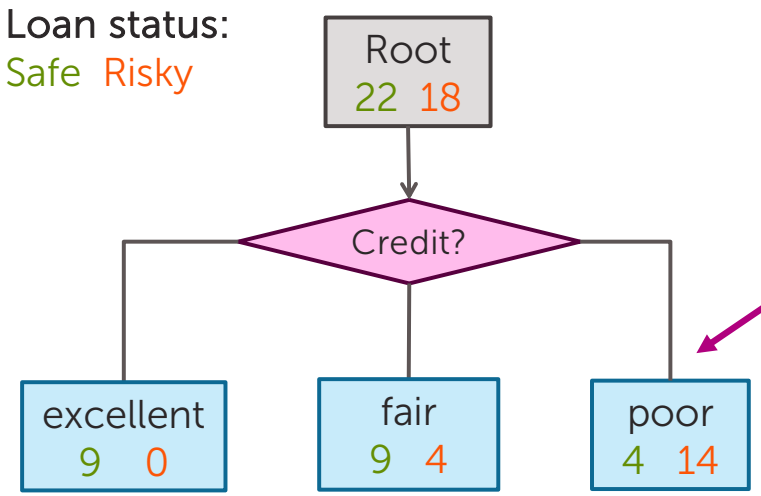
How do we learn a decision stump?



How do we select the best feature?



How do we measure effectiveness of a split?

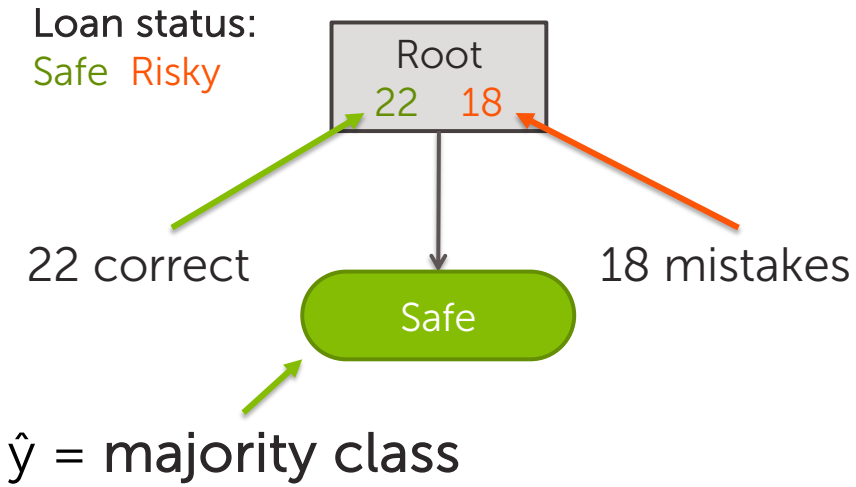


Idea: Calculate classification error of this decision stump

$$\text{Error} = \frac{\text{\# mistakes}}{\text{\# data points}}$$

Calculating classification error

- **Step 1:** $\hat{y}$ = class of majority of data in node
- **Step 2:** Calculate classification error of predicting $\hat{y}$ for this data



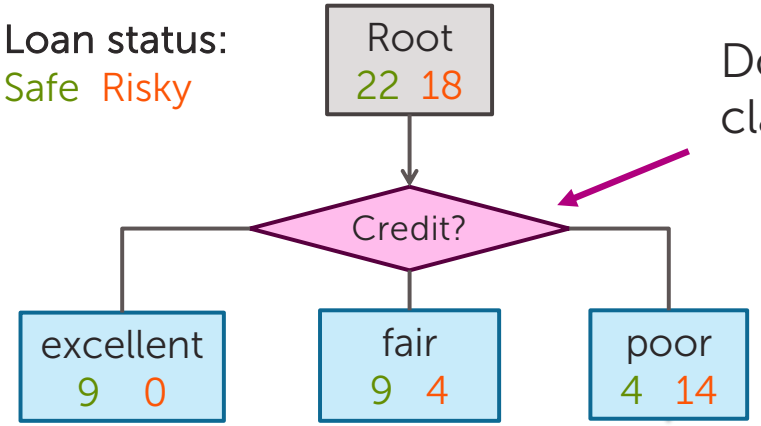
Error = _____

=

Tree	Classification error
(root)	0.45

Choice 1: Split on Credit history?

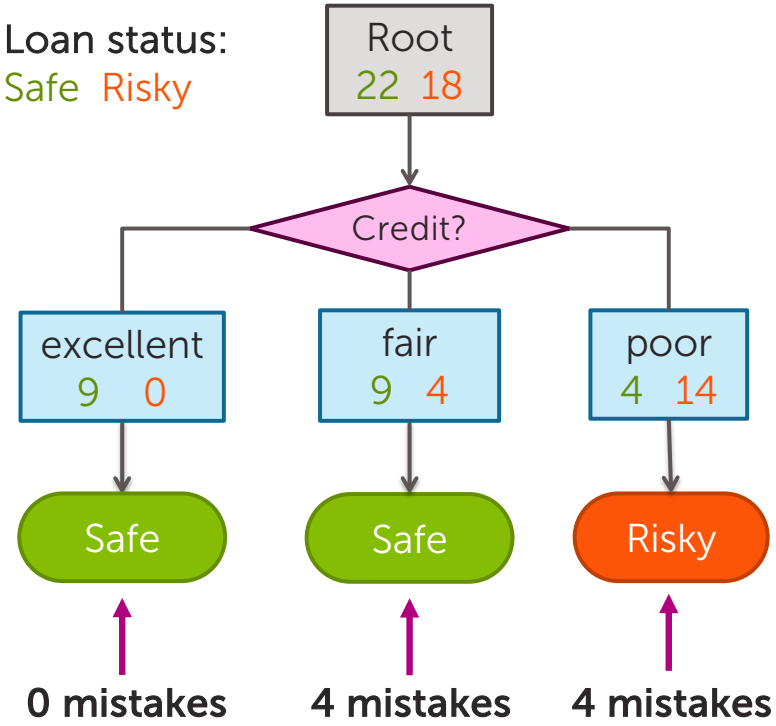
Choice 1: Split on Credit



Does a split on Credit reduce classification error below 0.45?

Split on Credit: Classification error

Choice 1: Split on Credit



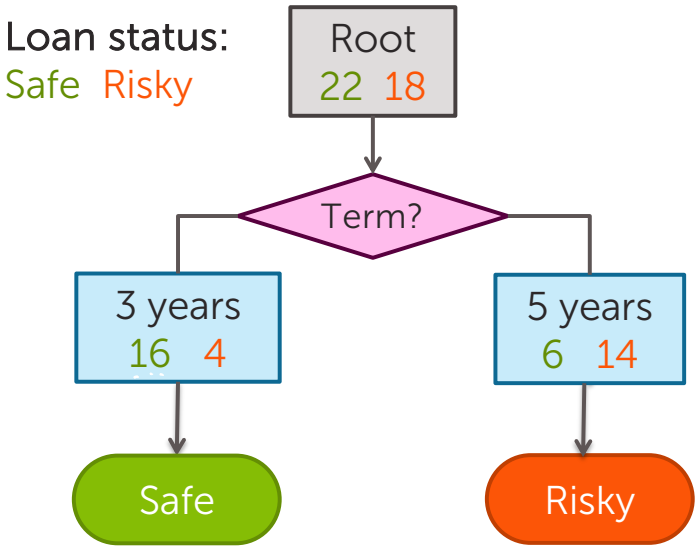
Error = _____

=

Tree	Classification error
(root)	0.45
Split on **credit**	0.2

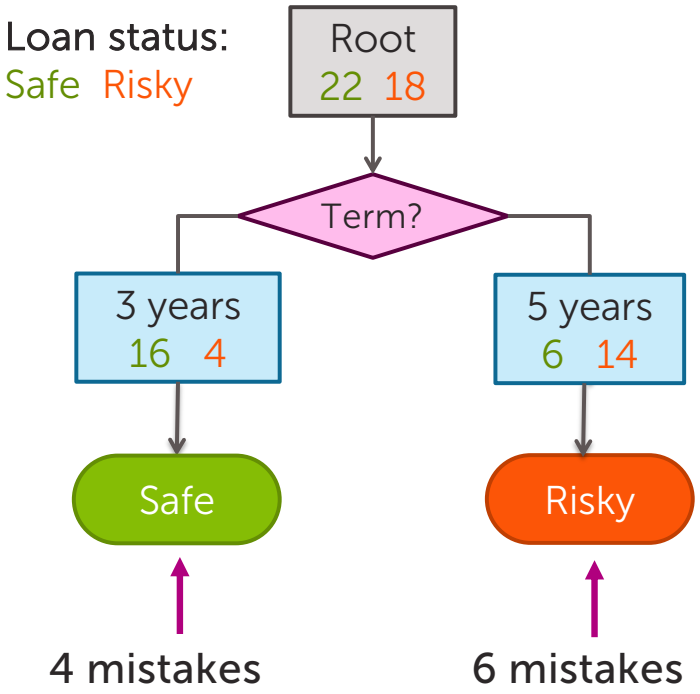
Choice 2: Split on Term?

Choice 2: Split on Term



Evaluating the split on Term

Choice 2: Split on Term



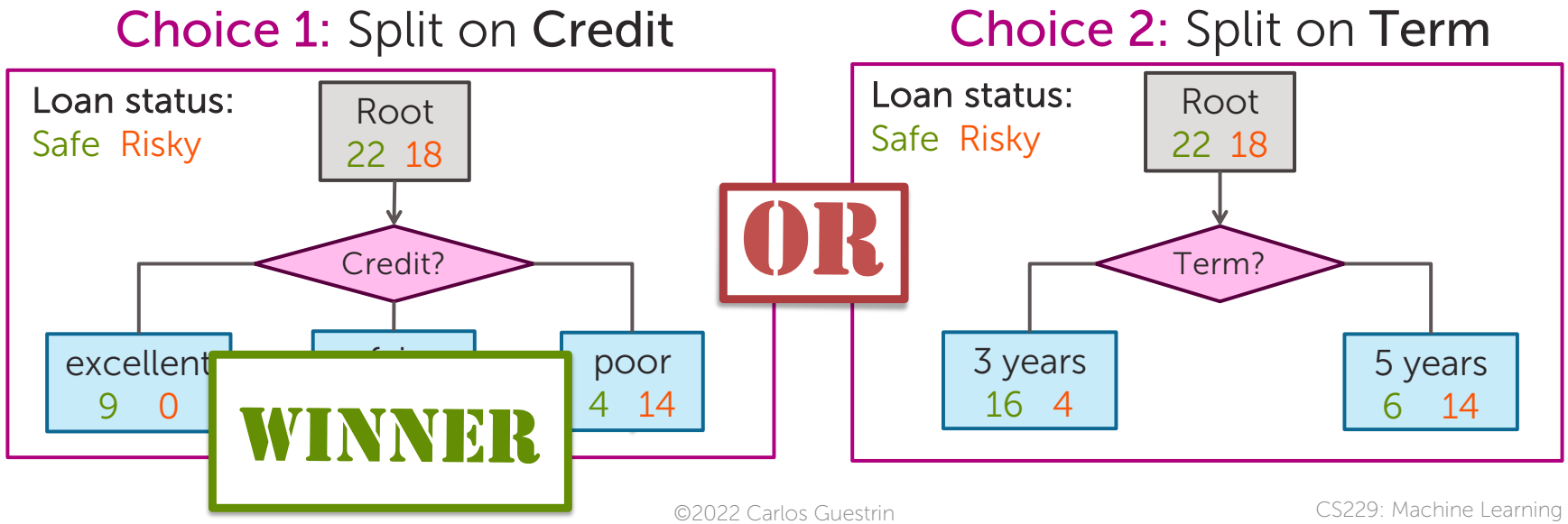
Error = _____

=

Tree	Classification error
(root)	0.45
Split on **credit**	0.2
Split on **term**	0.25

Choice 1 vs Choice 2: Comparing split on Credit vs Term

Tree	Classification error
(root)	0.45
split on **credit**	0.2
split on **loan term**	0.25

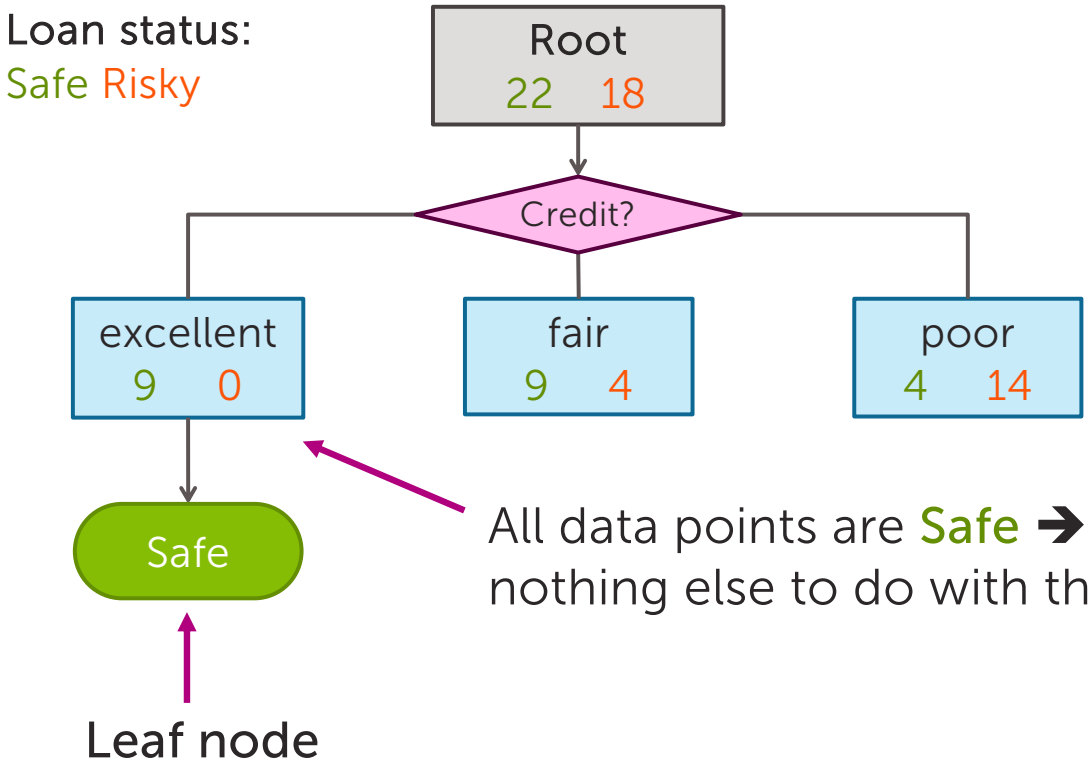


Feature split selection algorithm

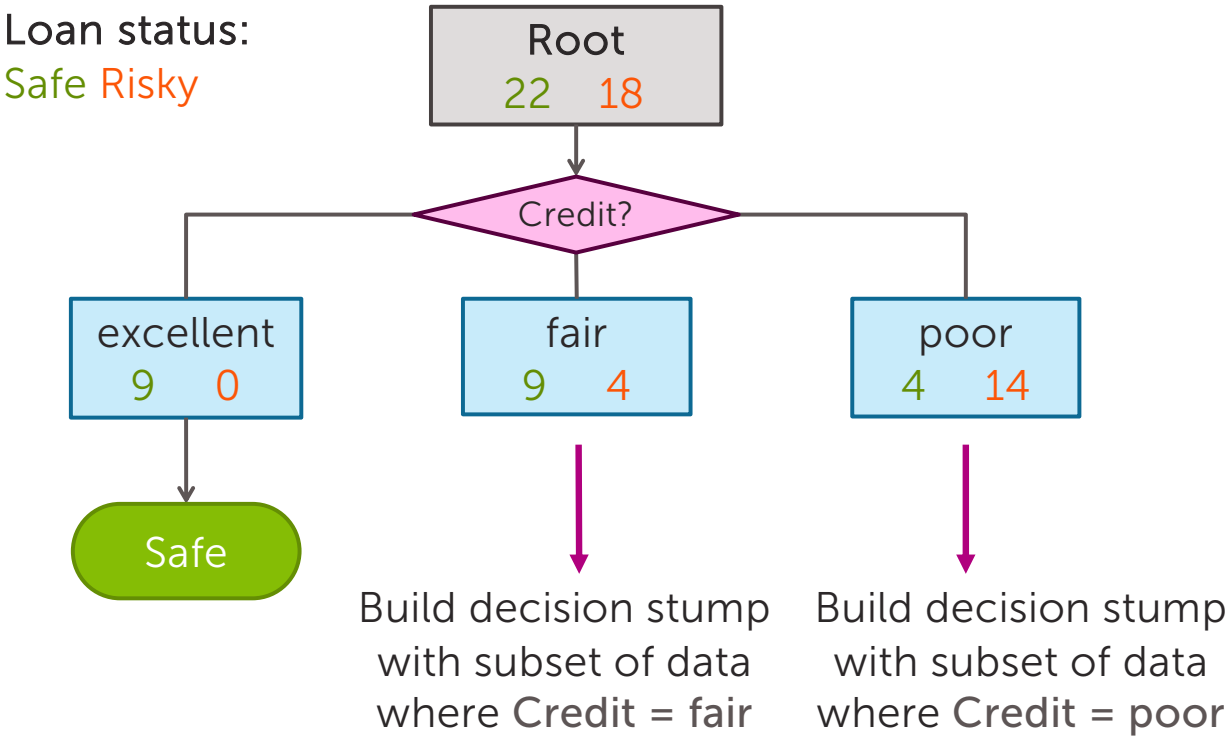
- Given a subset of data M (a node in a tree)
- For each feature $h_i(x)$:
  1. Split data of M according to feature $h_i(x)$
  2. Compute classification error of split
- Chose feature $h^*(x)$ with lowest classification error

Recursion & Stopping conditions

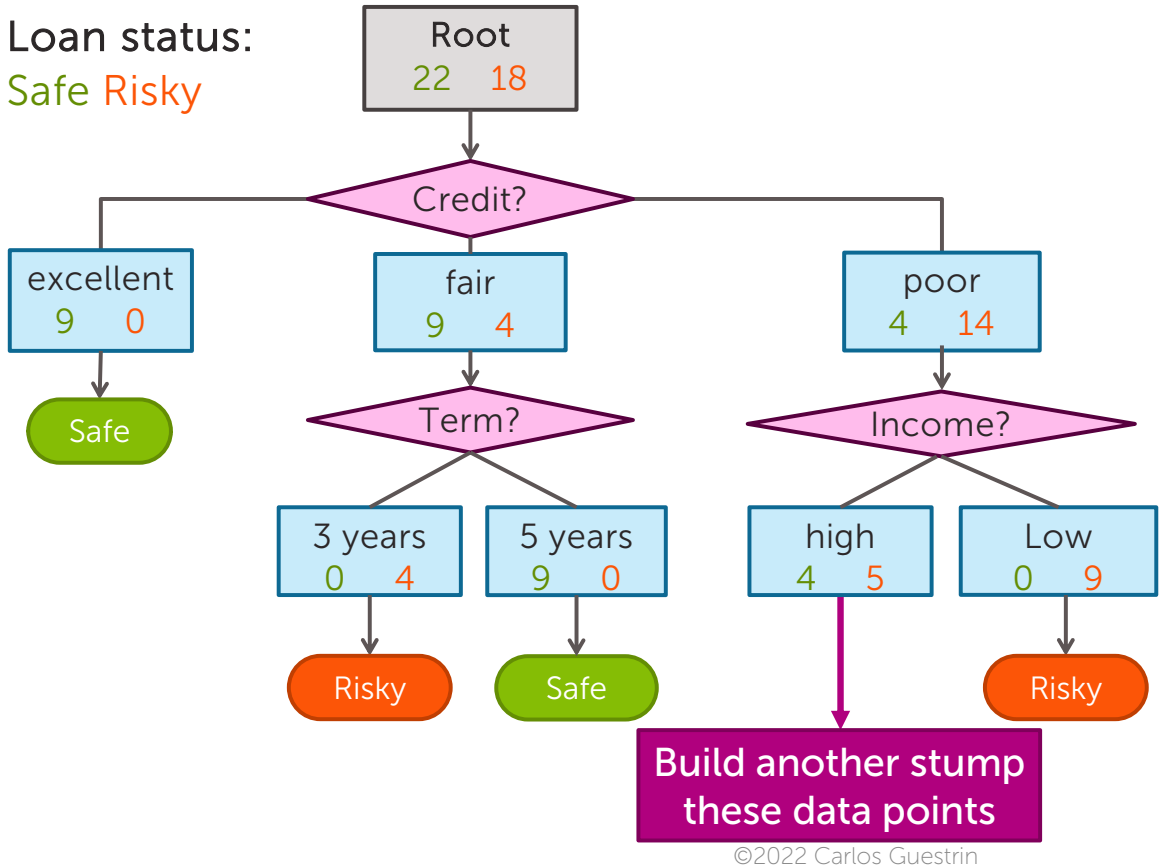
We've learned a decision stump, what next?



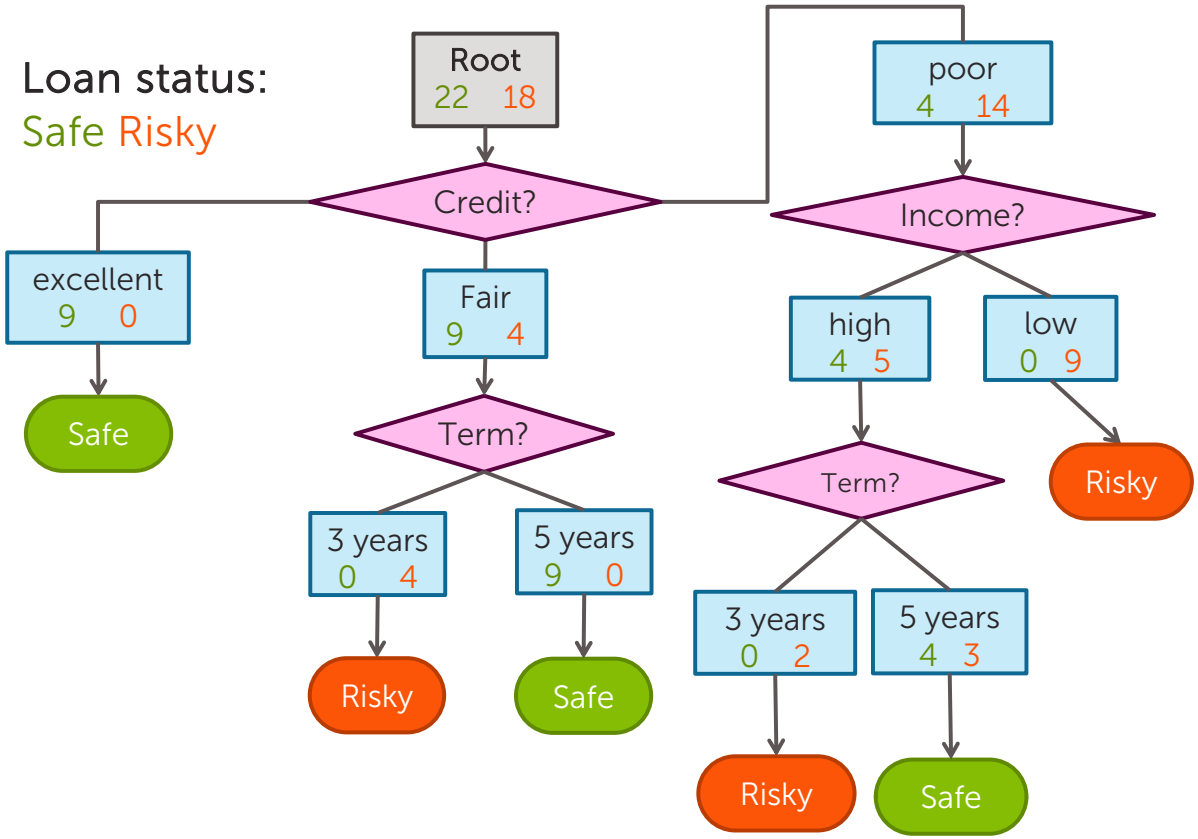
Tree learning = Recursive stump learning



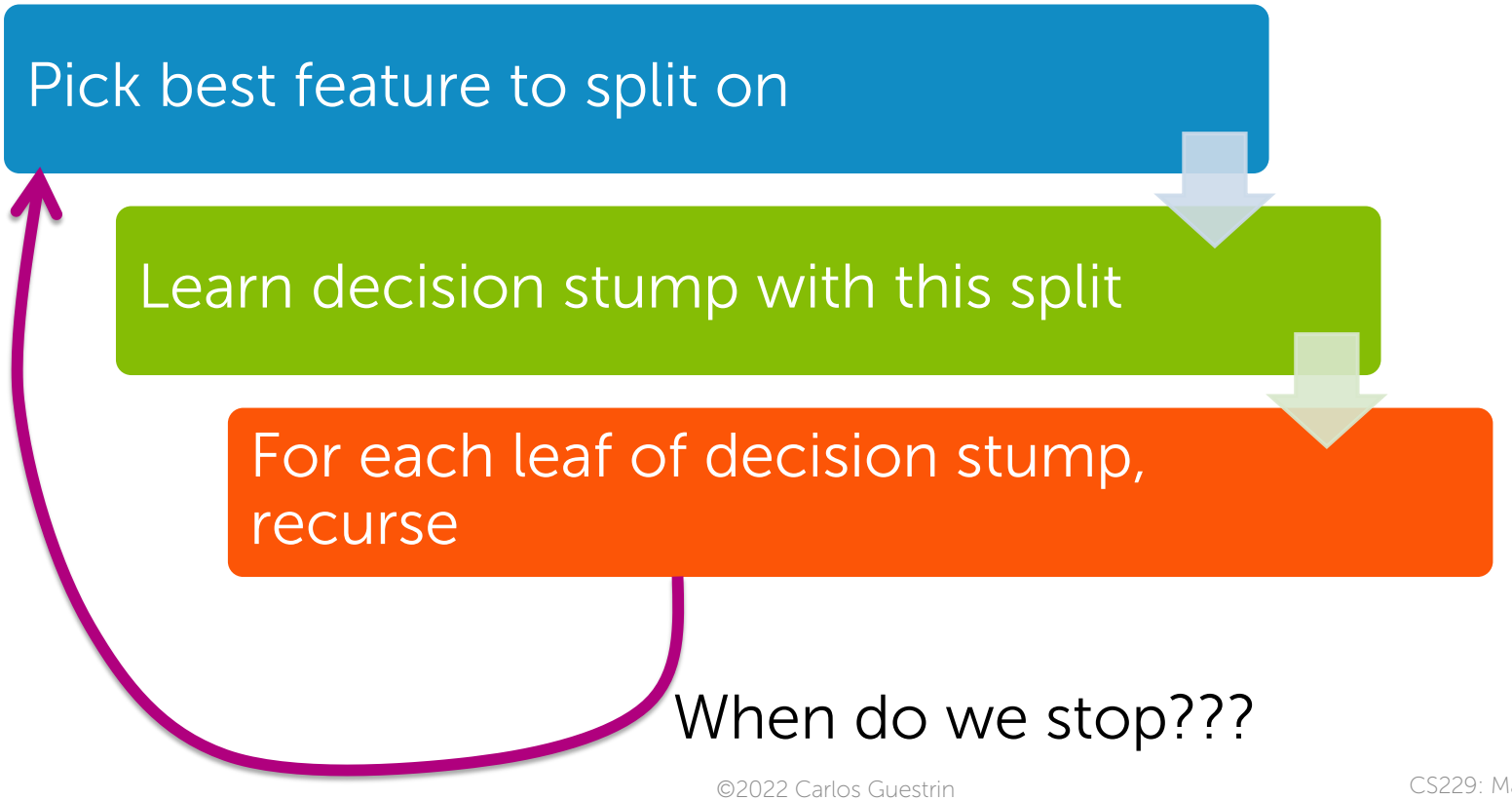
Second level



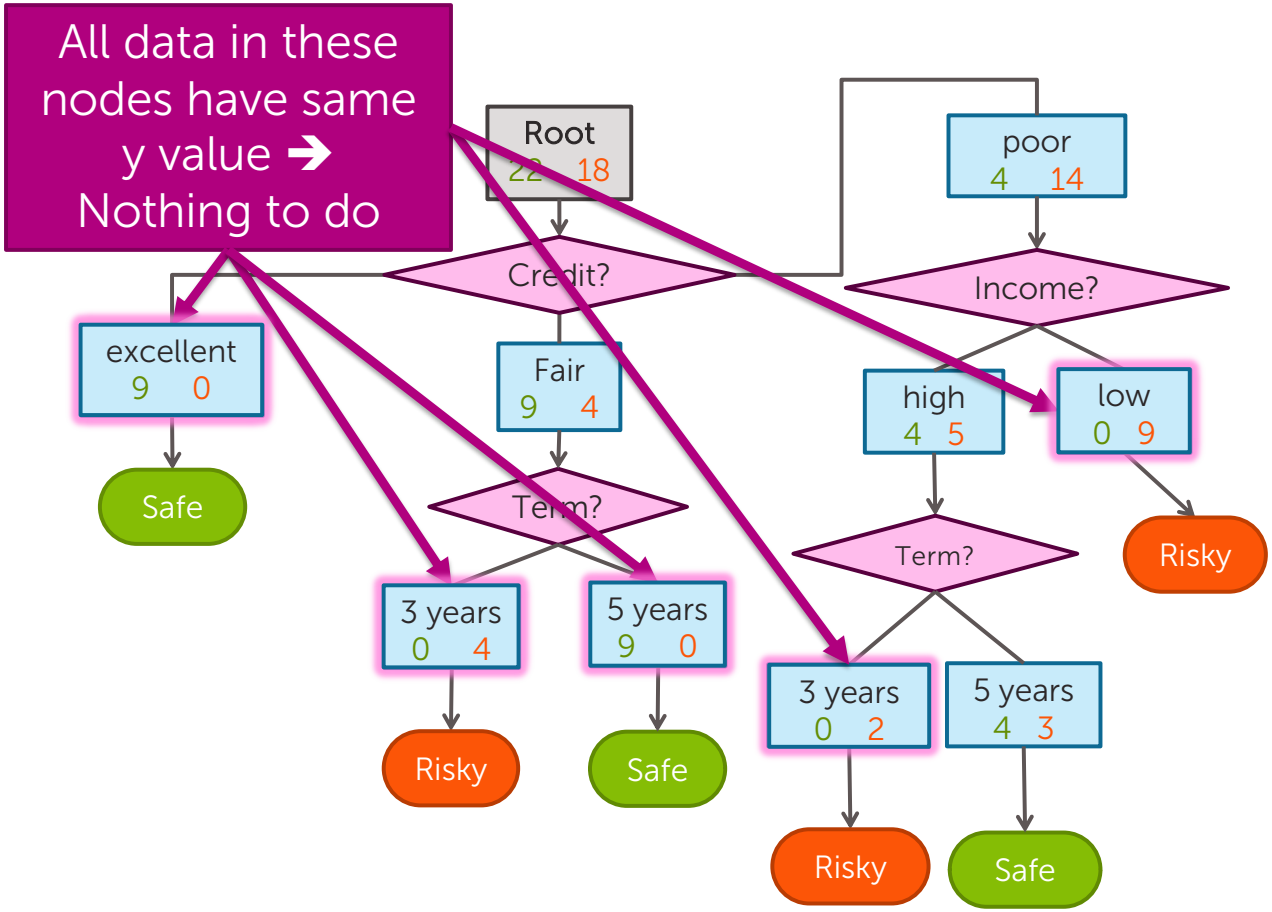
Final decision tree



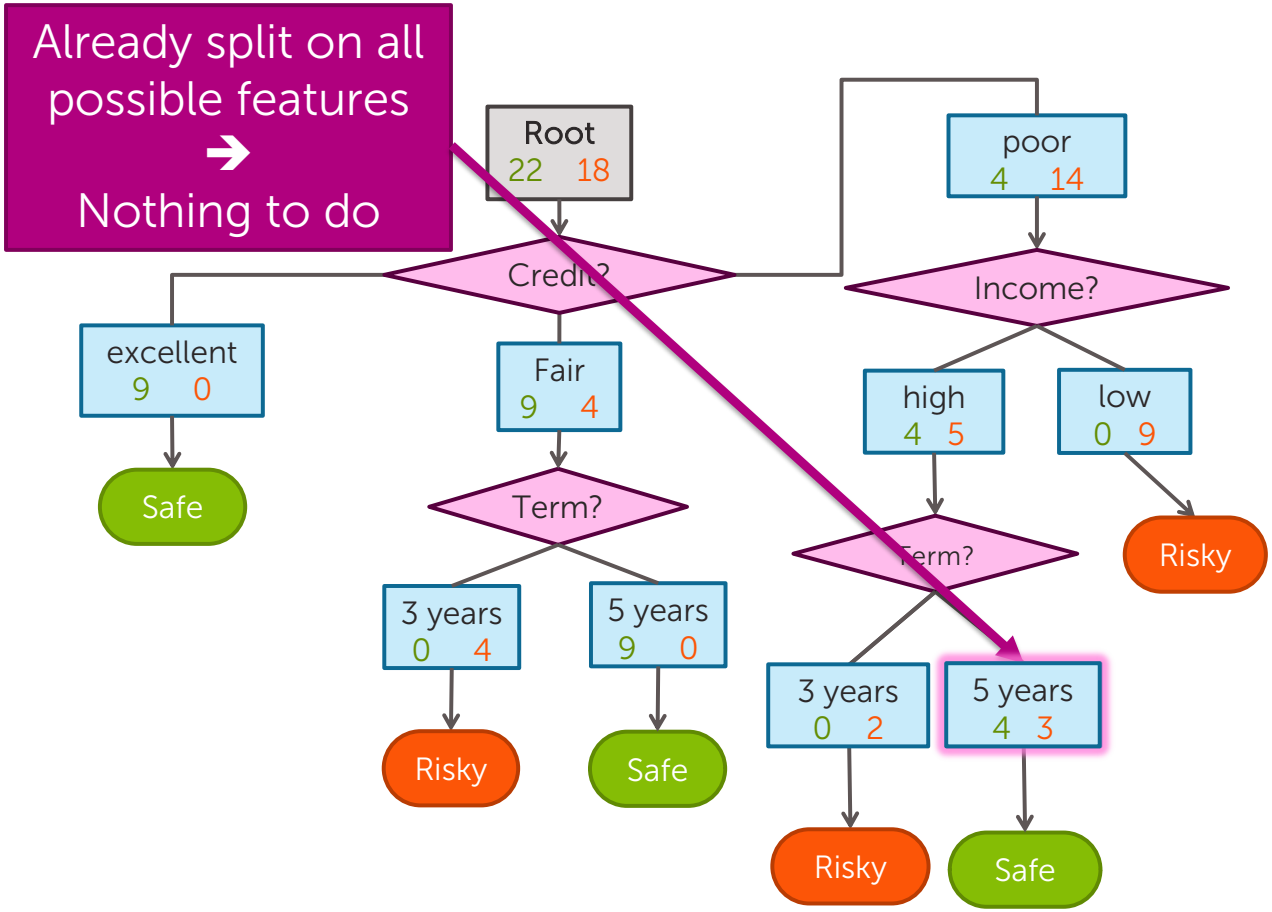
Simple greedy decision tree learning



Stopping condition 1: All data agrees on y

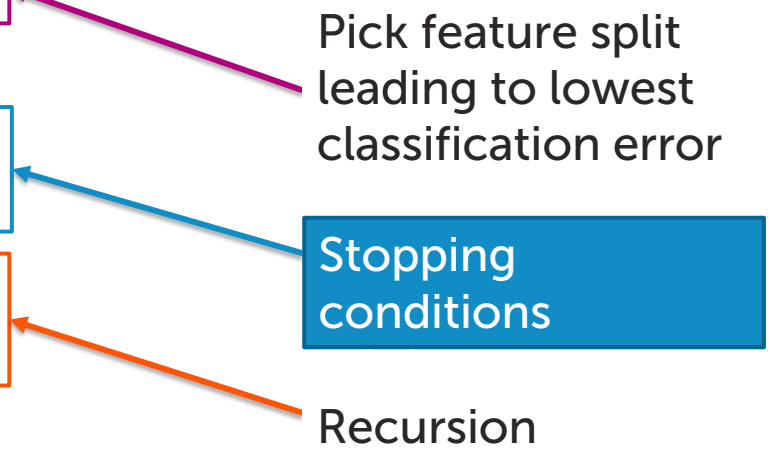


Stopping condition 2: Already split on all features

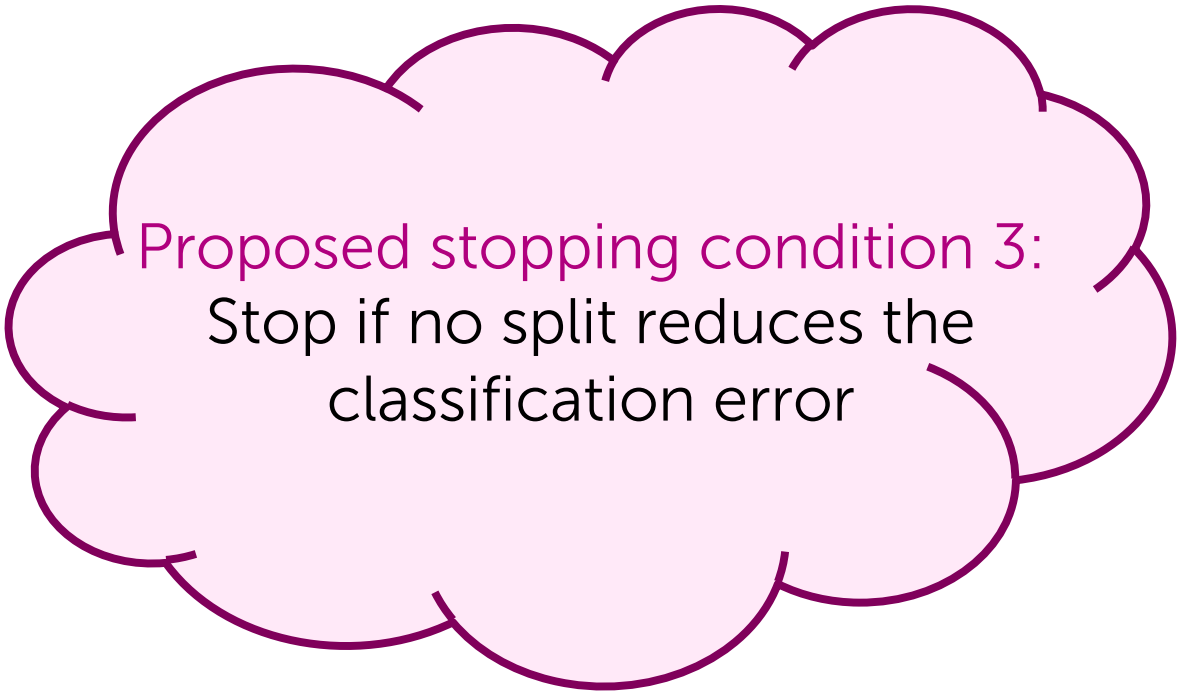


Greedy decision tree learning

- Step 1: Start with an empty tree
- Step 2: Select a feature to split data
- For each split of the tree:
  - Step 3: If nothing more to do, make predictions
  - Step 4: Otherwise, go to Step 2 & continue (recurse) on this split



Is this a good idea?



Stopping condition 3:
Don't stop if error doesn't decrease???

y = **x**[1] xor **x**[2]

x[1]	**x**[2]	y
False	False	False
False	True	True
True	False	True
True	True	False

y values
True False

Root
2 2

Error = _____

=

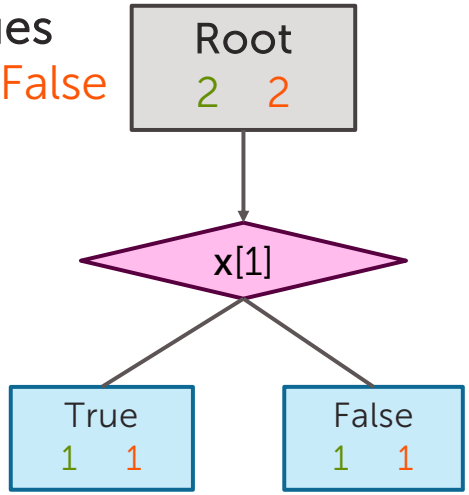
Tree	Classification error
(root)	0.5

Consider split on x[1]

y = **x**[1] xor **x**[2]

x[1]	**x**[2]	y
False	False	False
False	True	True
True	False	True
True	True	False

y values
True False



Error = _____

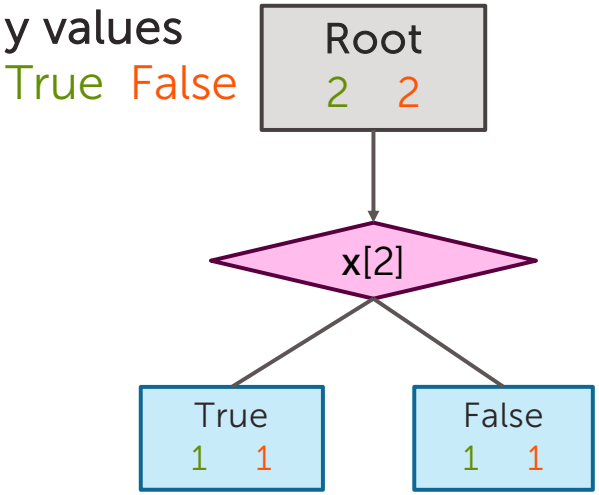
=

Tree	Classification error
(root)	0.5
Split on **x**[1]	0.5

Consider split on x[2]

$y = x[1]$ xor $x[2]$

x[1]	x[2]	y
False	False	False
False	True	True
True	False	True
True	True	False



$$\text{Error} = \frac{1+1}{2+2} = 0.5$$

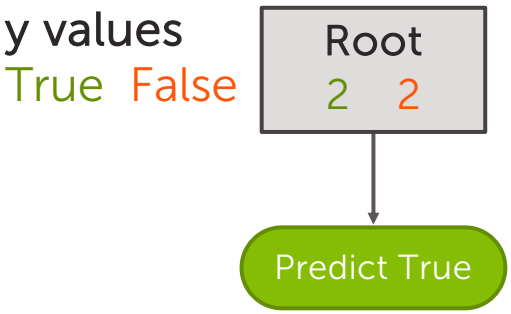
Neither features improve training error... Stop now???

Tree	Classification error
(root)	0.5
Split on x[1]	0.5
Split on x[2]	0.5

Final tree with stopping condition 3

$y = x[1]$ xor $x[2]$

x[1]	x[2]	y
False	False	False
False	True	True
True	False	True
True	True	False



Tree	Classification error
with stopping condition 3	0.5

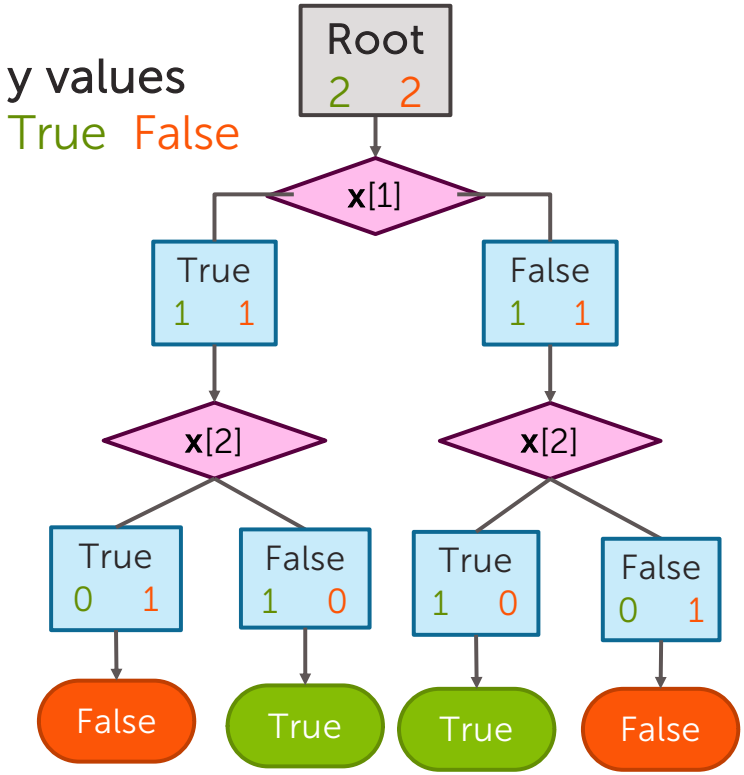
Without stopping condition 3

Condition 3 (stopping when training error doesn't' improve) is **not** recommended!

$y = x[1]$ xor $x[2]$

x[1]	x[2]	y
False	False	False
False	True	True
True	False	True
True	True	False

Tree	Classification error
with stopping condition 3	0.5
without stopping condition 3	

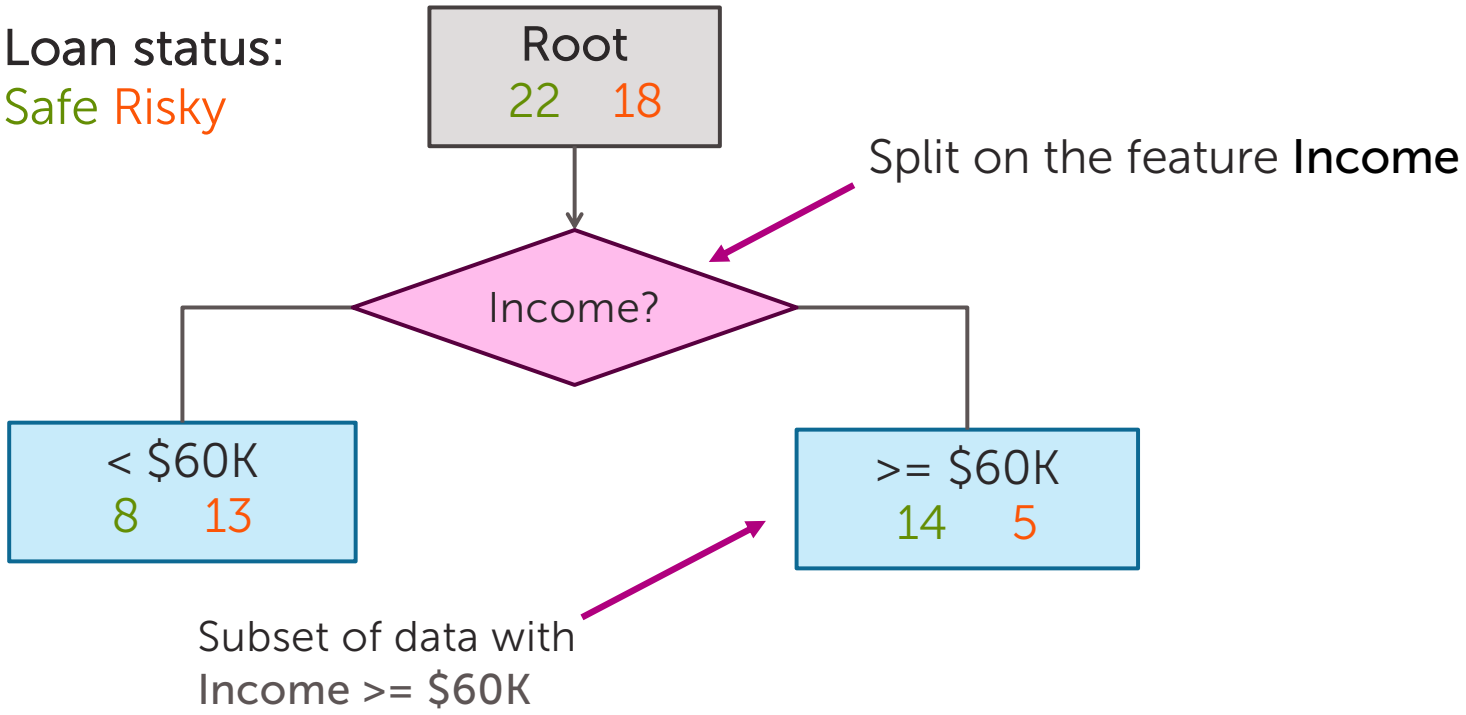


Decision tree learning:
Real valued features

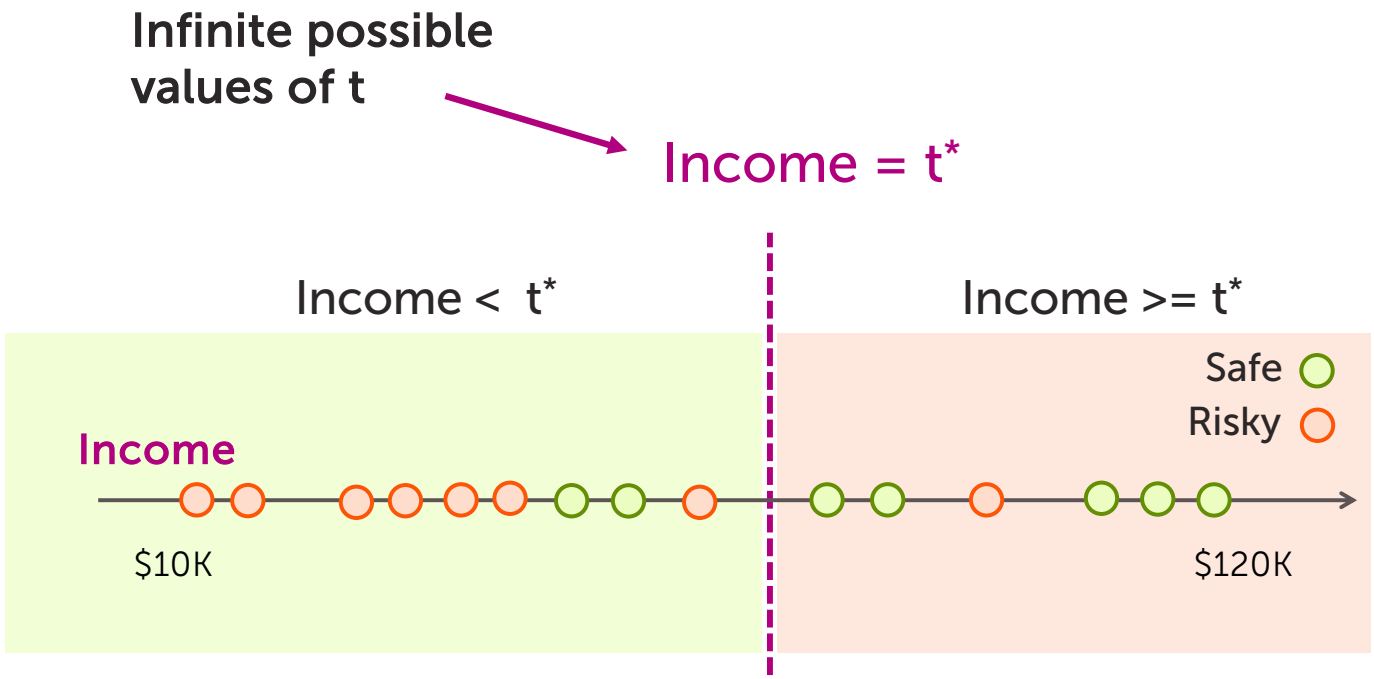
How do we use real values inputs?

Income	Credit	Term	y
$105 K	excellent	3 yrs	Safe
$112 K	good	5 yrs	Risky
$73 K	fair	3 yrs	Safe
$69 K	excellent	5 yrs	Safe
$217 K	excellent	3 yrs	Risky
$120 K	good	5 yrs	Safe
$64 K	fair	3 yrs	Risky
$340 K	excellent	5 yrs	Safe
$60 K	good	3 yrs	Risky

Threshold split

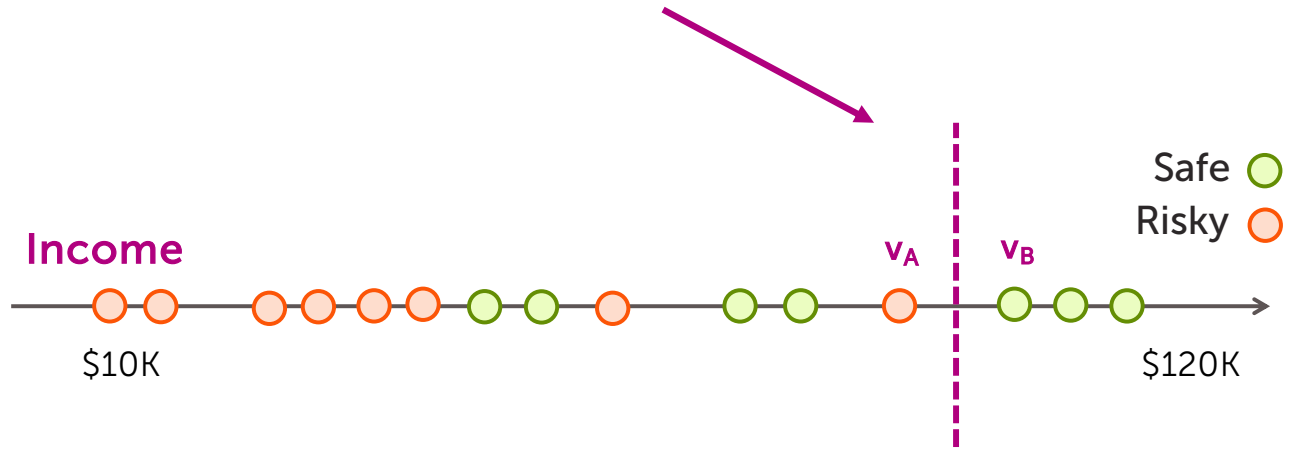


Finding the best threshold split

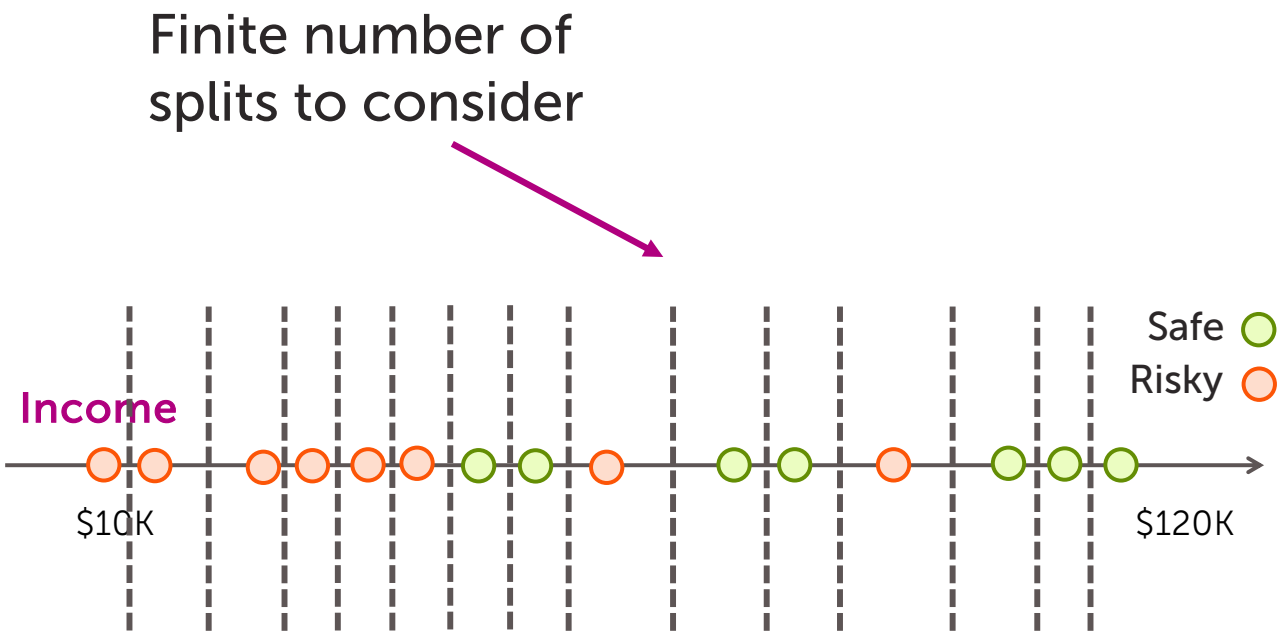


Consider a threshold between points

Same classification error for any threshold split between v_A and v_B



Only need to consider mid-points



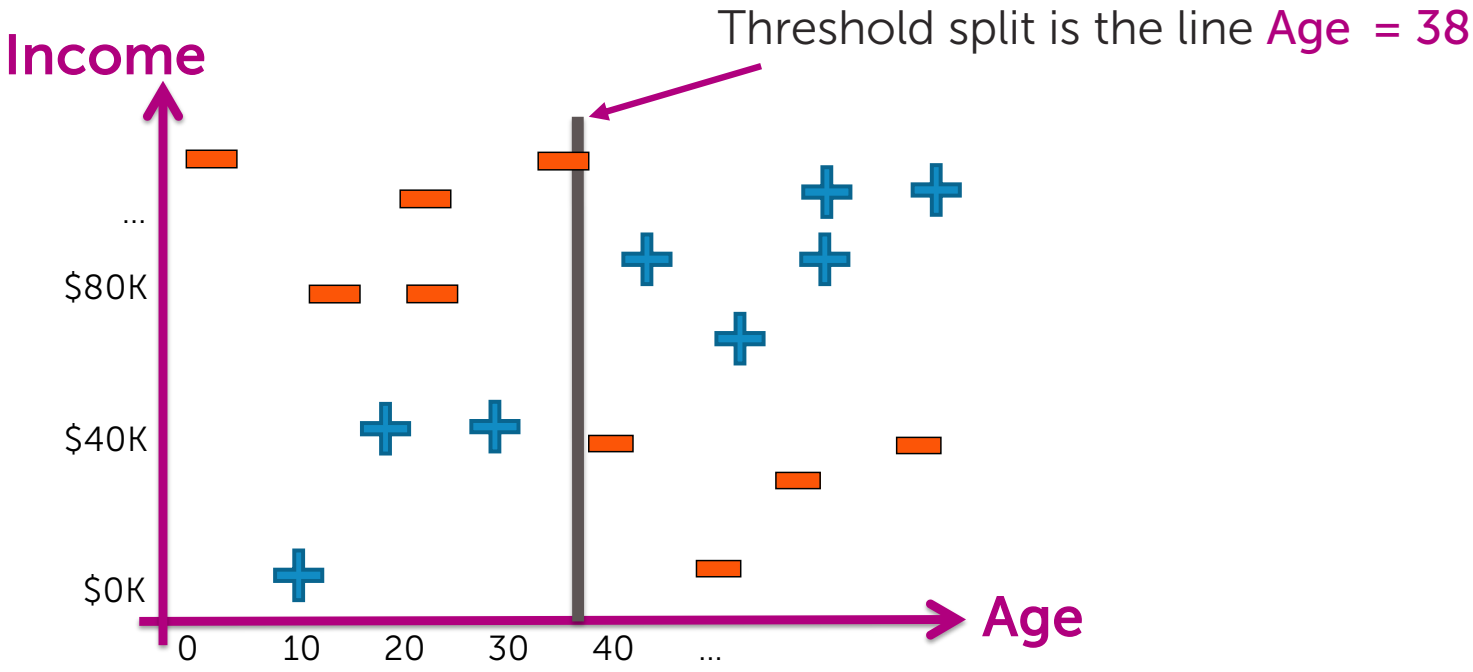
Threshold split selection algorithm

- Step 1: Sort the values of a feature $h_j(\mathbf{x})$:

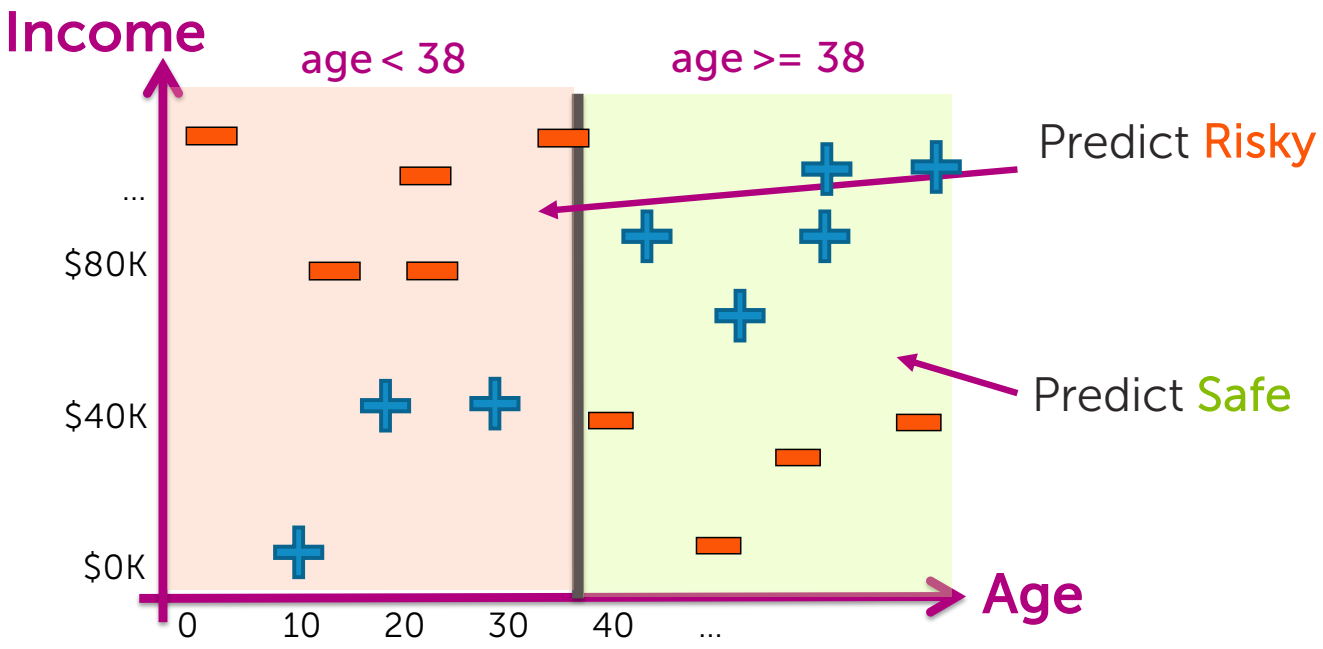
  Let $\{v_1, v_2, v_3, \ldots v_N\}$ denote sorted values

- Step 2:
  - For i = 1 … N-1
    - Consider split $t_i = (v_i + v_{i+1}) / 2$
    - Compute classification error for treshold split $h_j(\mathbf{x}) >= t_i$
  - Chose the t^* with the lowest classification error

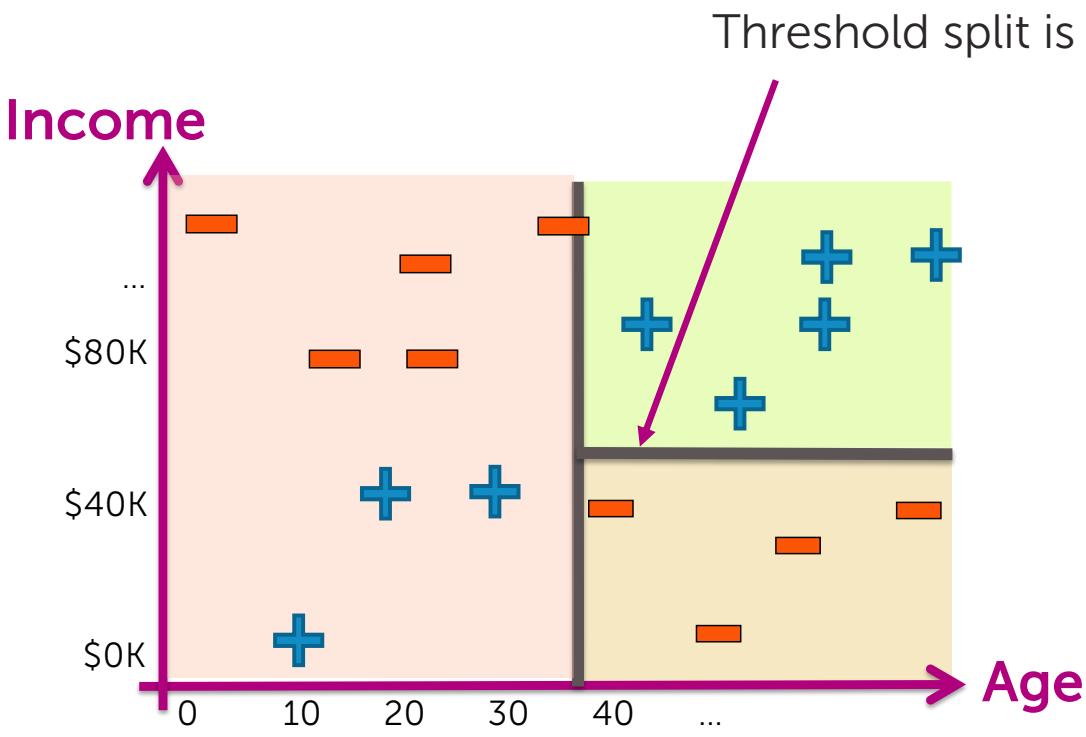
Visualizing the threshold split



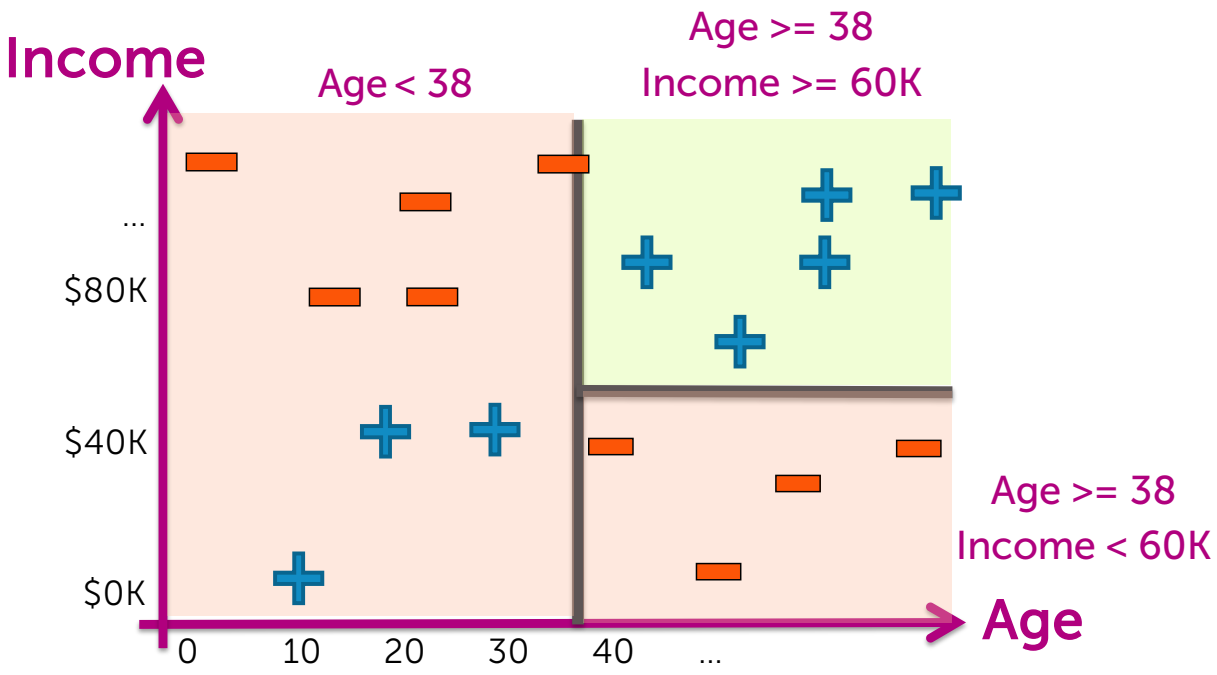
Split on Age >= 38



Depth 2: Split on Income >= $60K



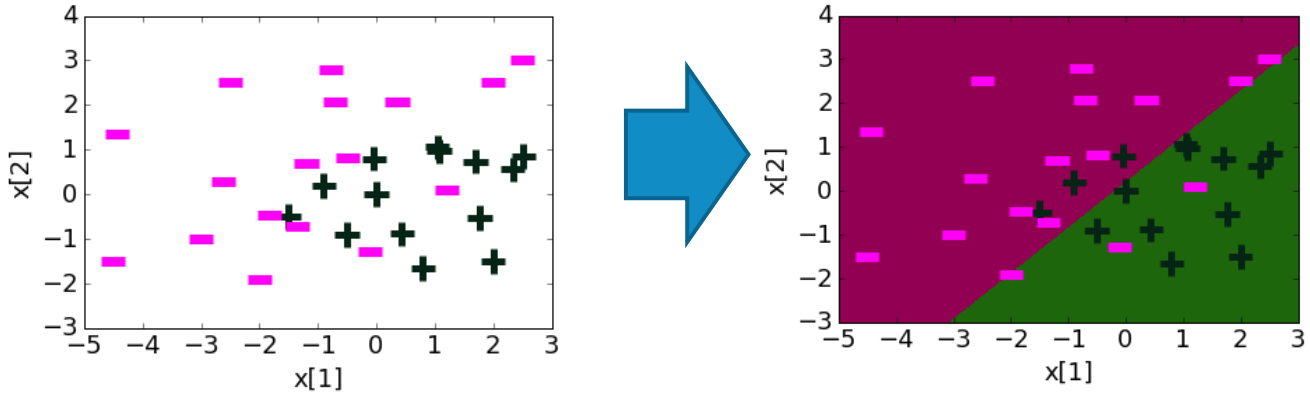
Each split partitions the 2-D space



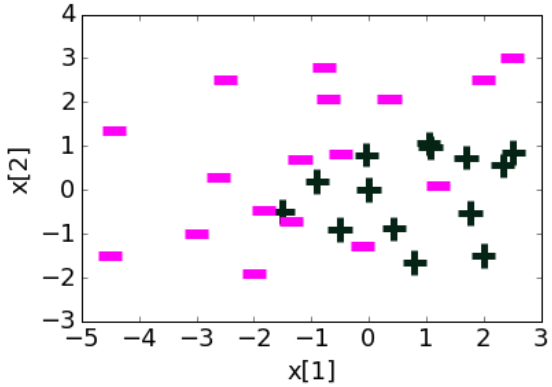
Decision trees vs logistic regression: *Example*

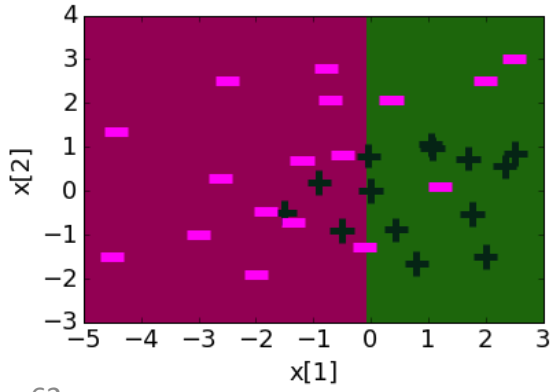
Logistic regression

Feature	Value	Weight Learned
$h_0(x)$	1	0.22
$h_1(x)$	x[1]	1.12
$h_2(x)$	x[2]	-1.07

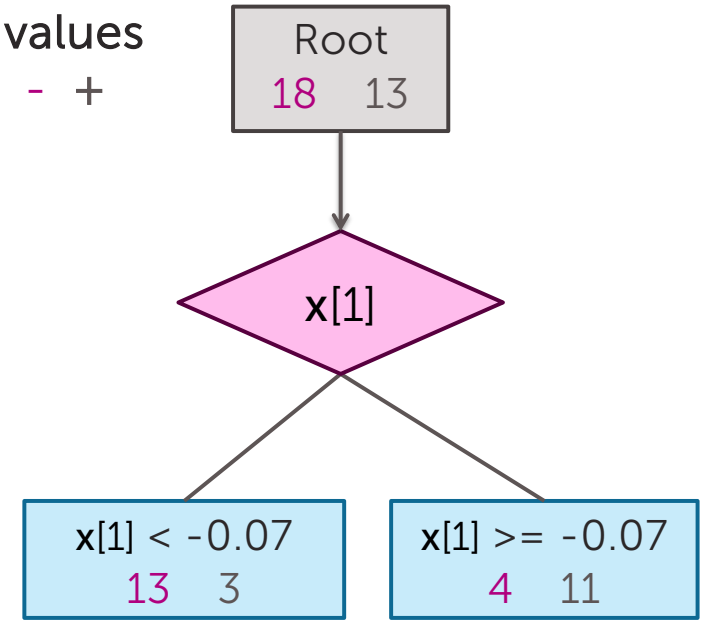


Depth 1: Split on x[1]

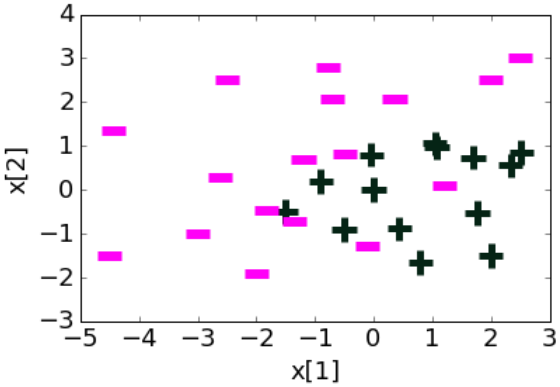


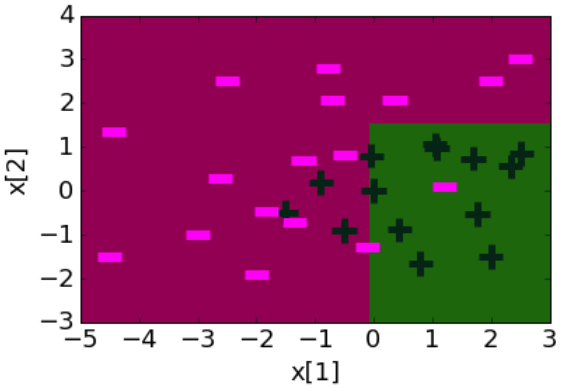


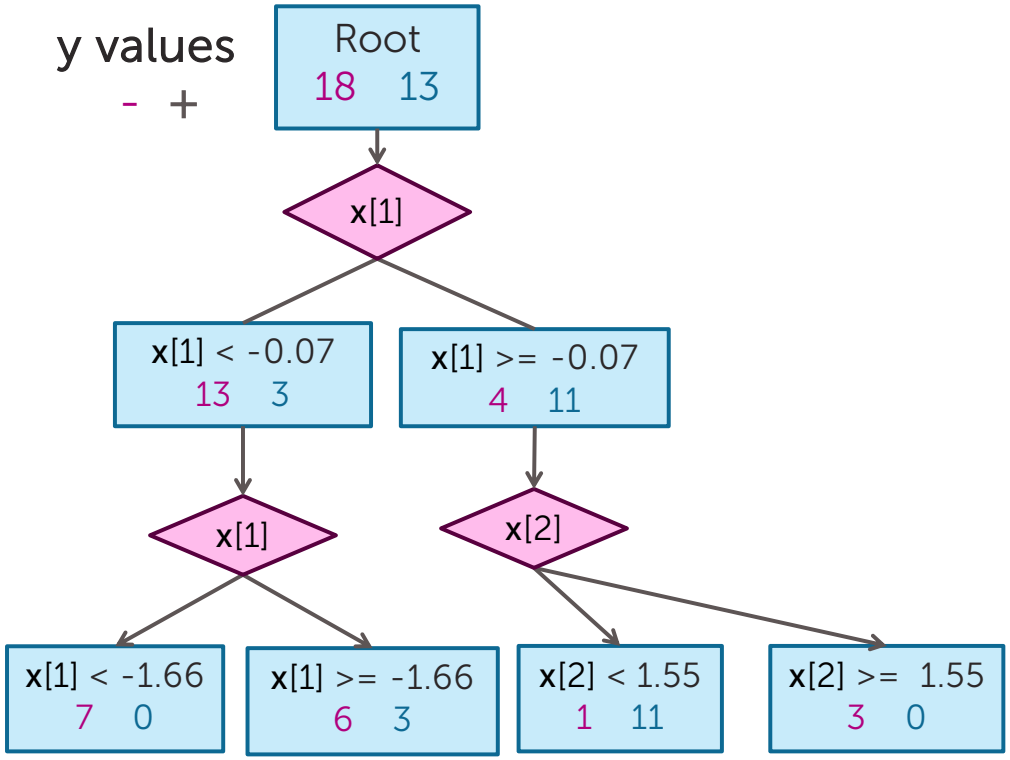
y values
- +



Depth 2

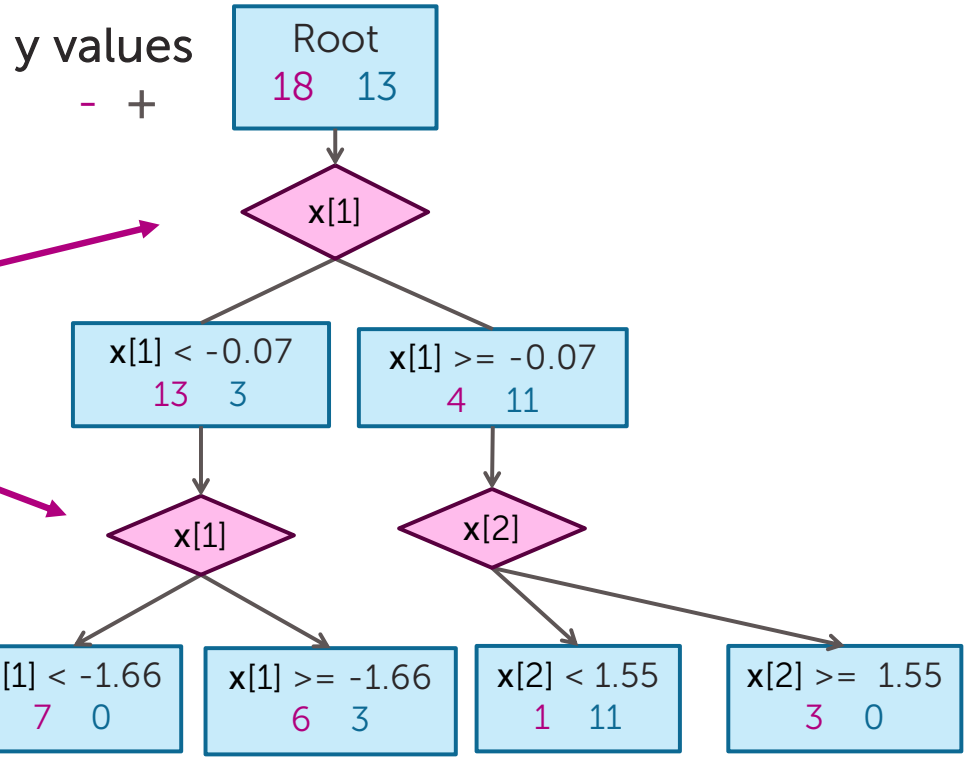




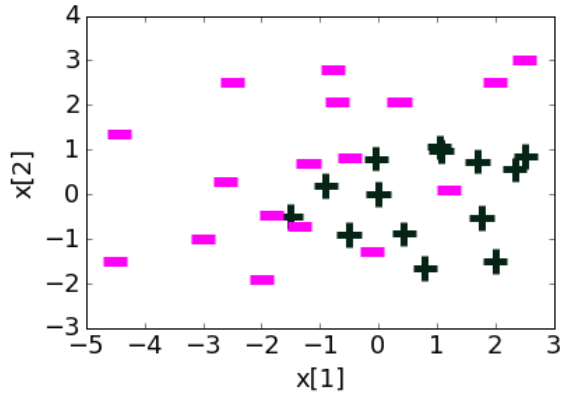


Threshold split caveat

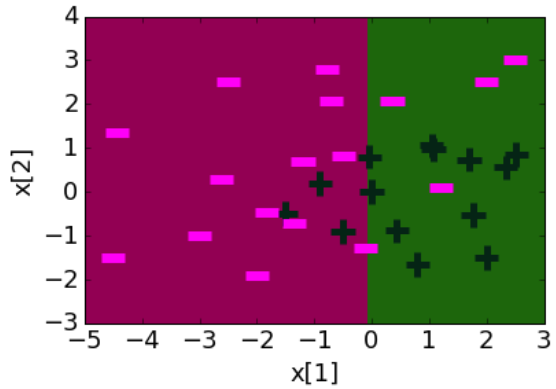
For threshold splits, same feature can be used multiple times



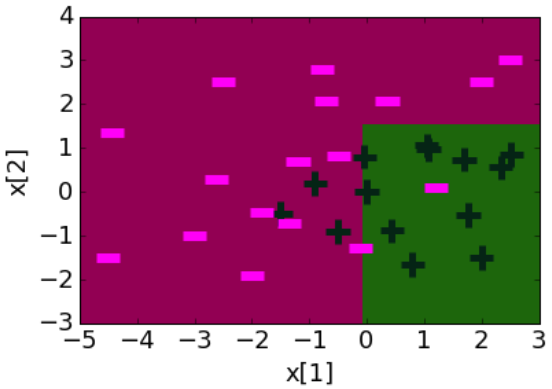
Decision boundaries



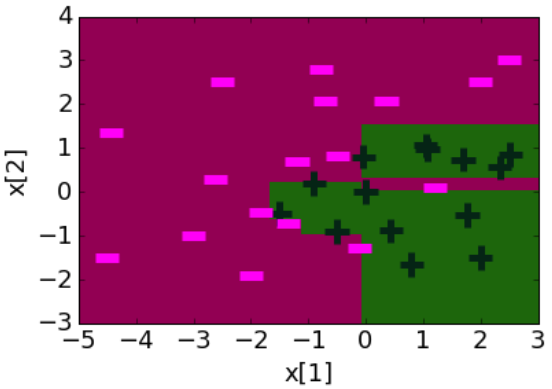
Depth 1



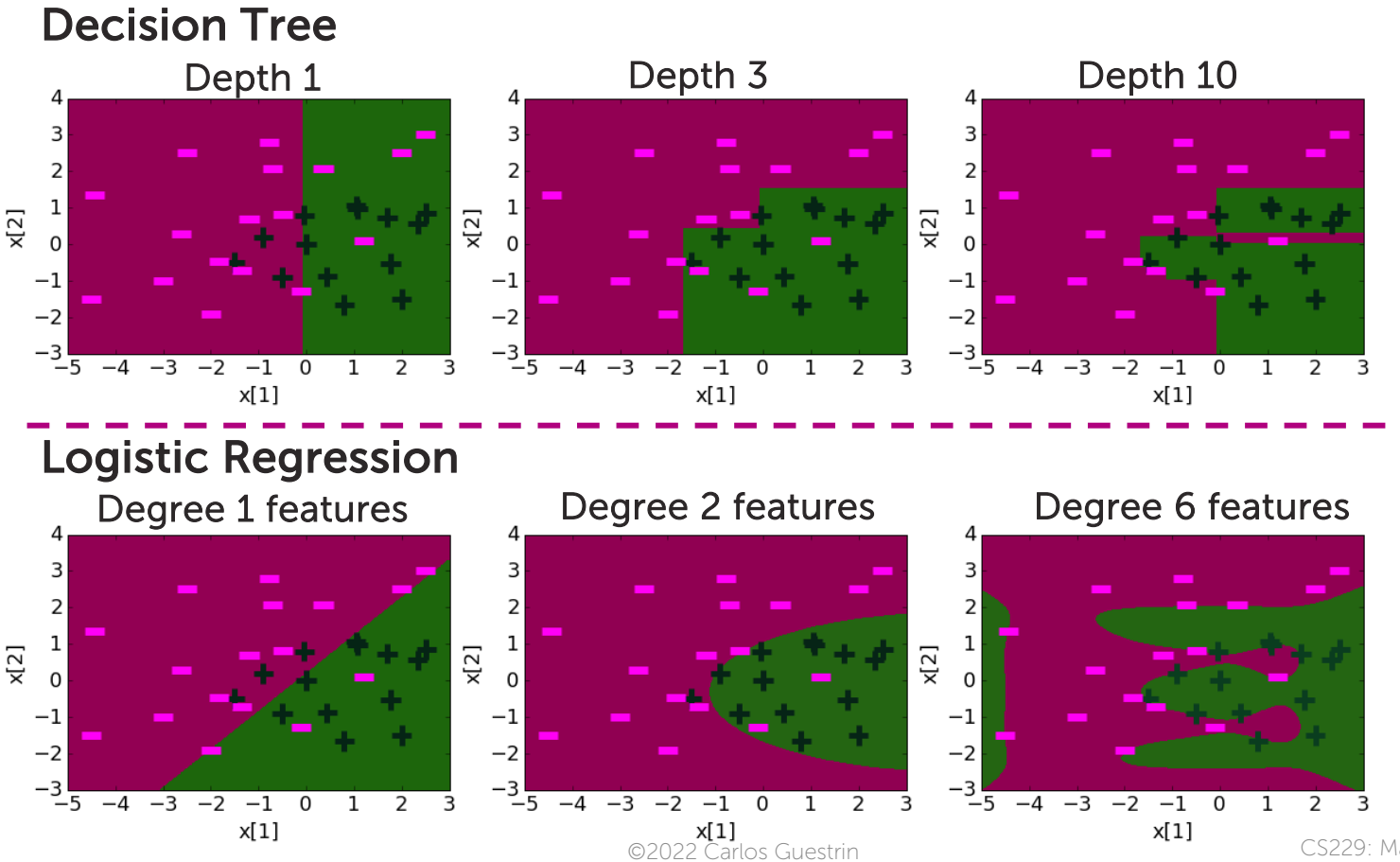
Depth 2



Depth 10



Comparing decision boundaries



Summary of decision trees

What you can do now

- Define a decision tree classifier
- Interpret the output of a decision trees
- Learn a decision tree classifier using greedy algorithm
- Traverse a decision tree to make predictions
  - Majority class predictions
- Tackle continuous and discrete features