Decision Trees

CS229: Machine Learning
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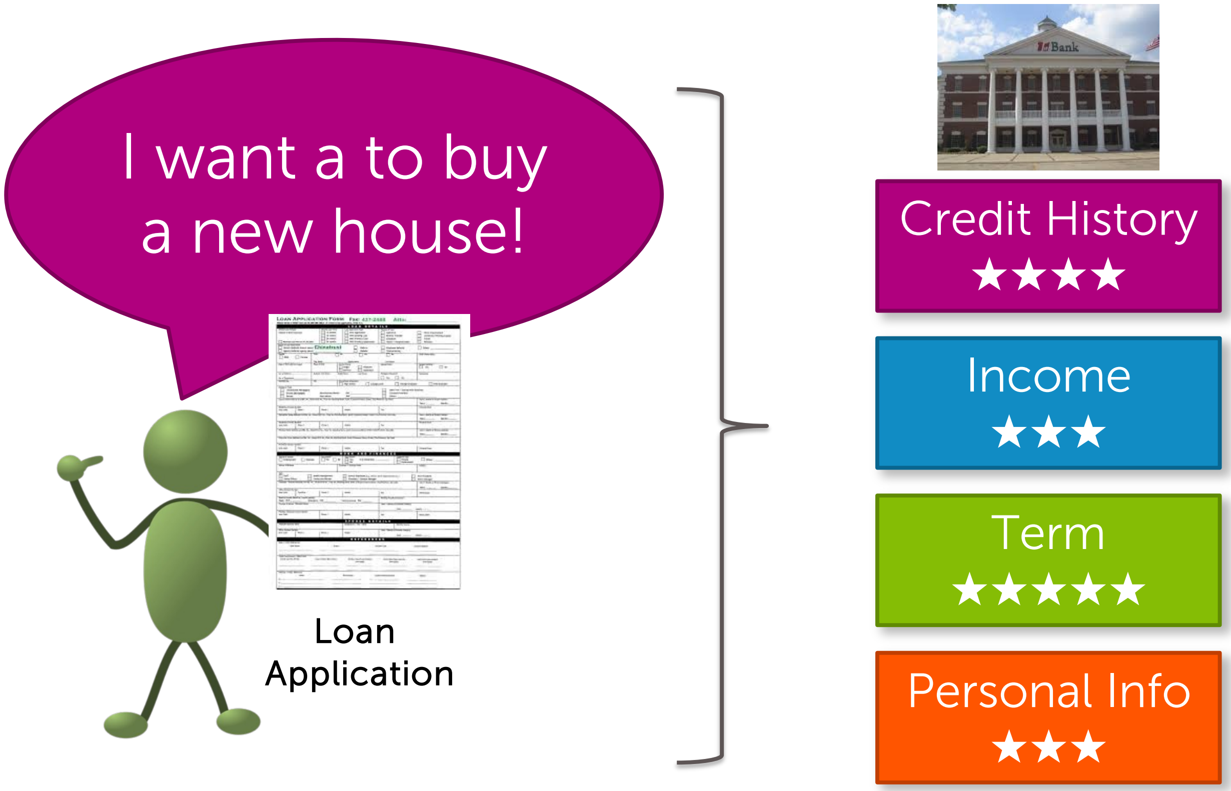
Stanford University

Slides include content developed by and co-developed with Emily Fox

Predicting potential loan defaults

What makes a loan risky?



I want a to buy a new house!

Loan Application

Credit History ★★★★

Income ★★★

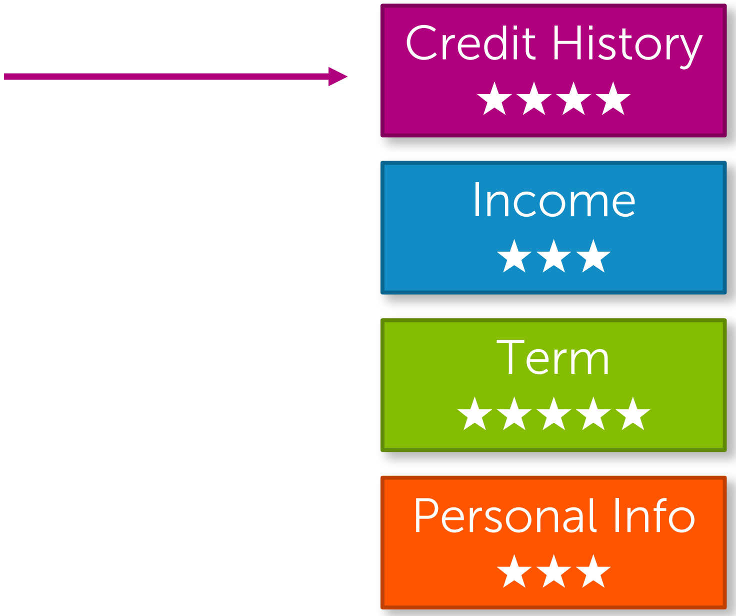
Term ★★★★★

Personal Info ★★★

Credit history explained

Did I pay previous loans on time?

Example:
excellent, good, or fair



Credit History
★★★★

Income
★★★

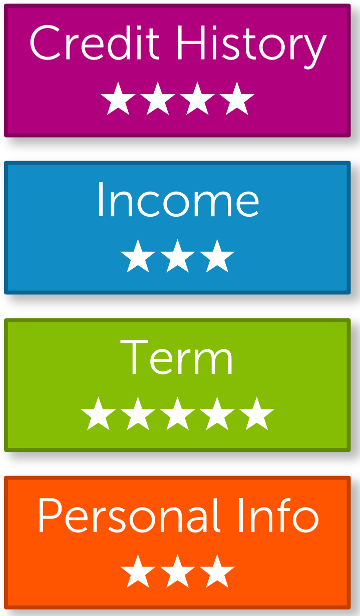
Term
★★★★★

Personal Info
★★★

Income

What's my income?

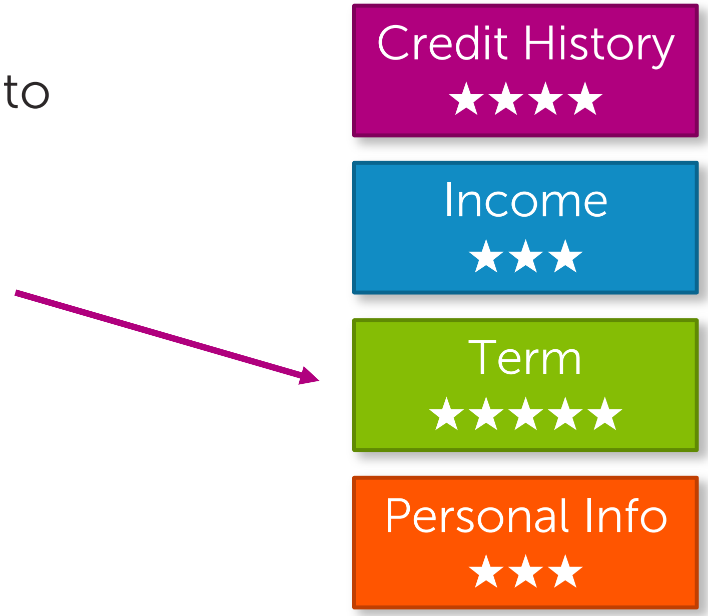
Example:
$80K per year



Loan terms

How soon do I need to pay the loan?

Example: 3 years, 5 years,…



Credit History

★★★★

Income

★★★

Term

★★★★★

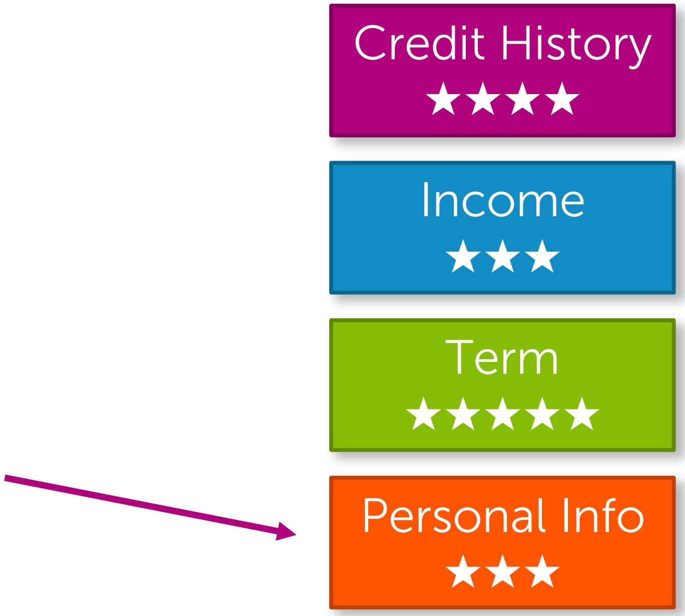
Personal Info

★★★

Personal information

Age, reason for the loan, marital status,…

Example: Home loan for a married couple



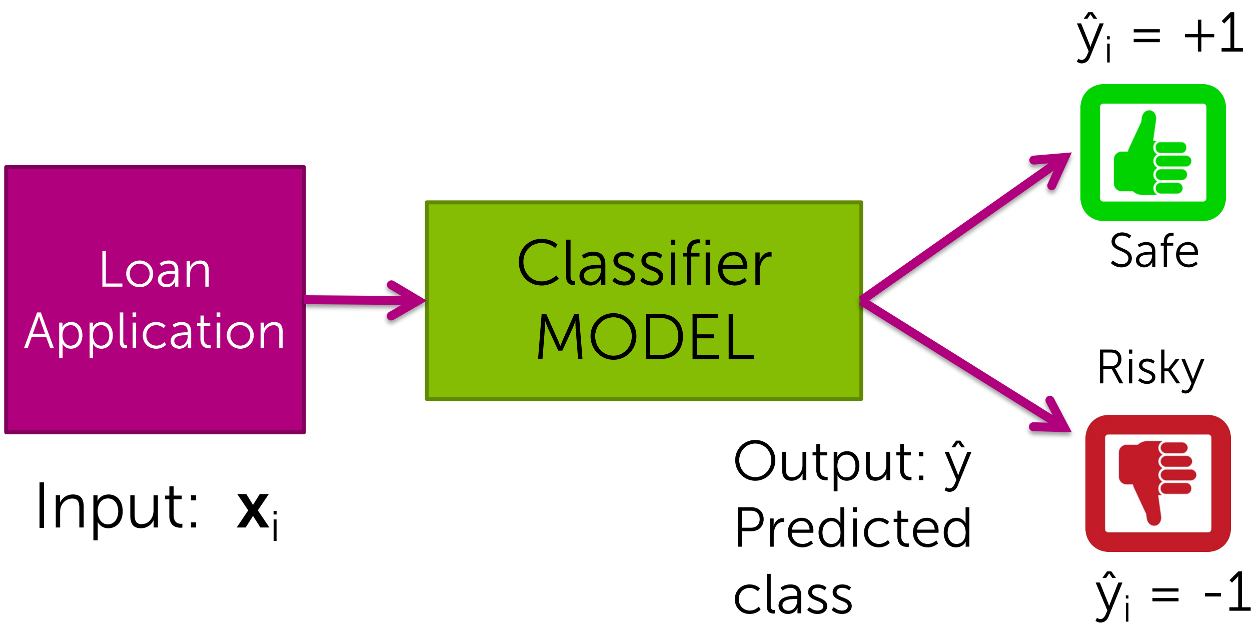
Credit History ★★★★

Income ★★★

Term ★★★★★

Personal Info ★★★

Classifier review



Loan Application

Input: $\mathbf{x}_i$

Classifier MODEL

Output: $\hat{y}$

Predicted class

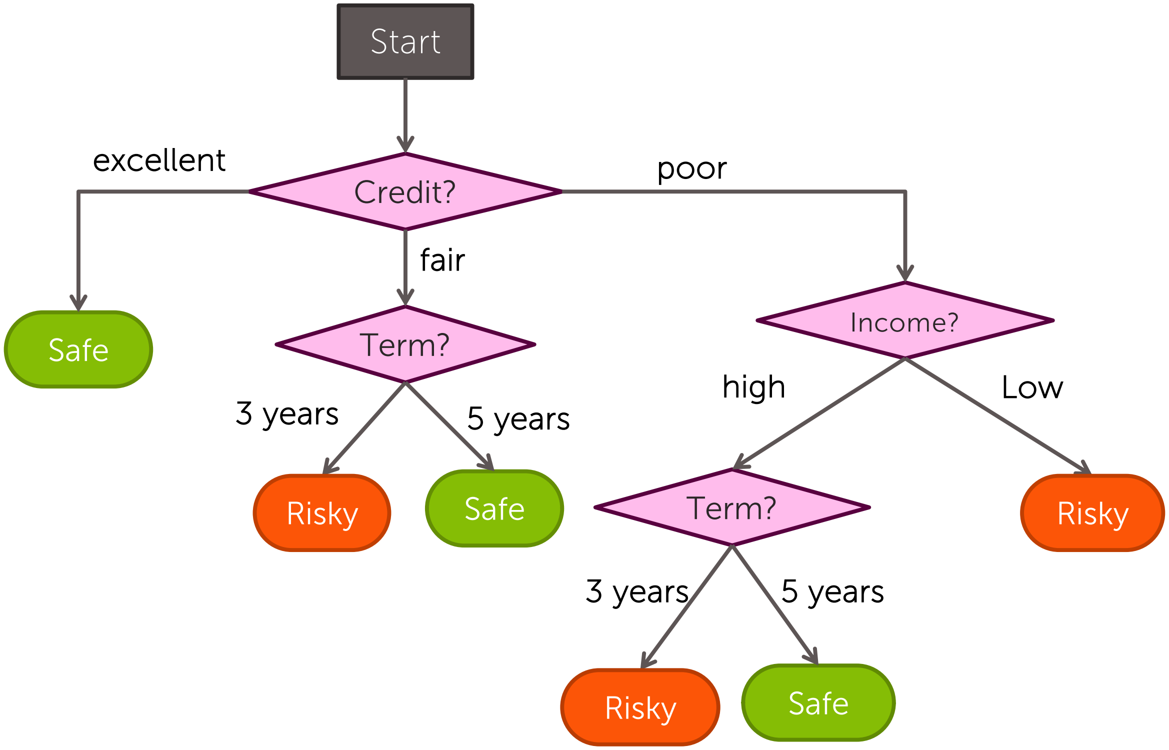
$\hat{y}_i = +1$

Safe

Risky

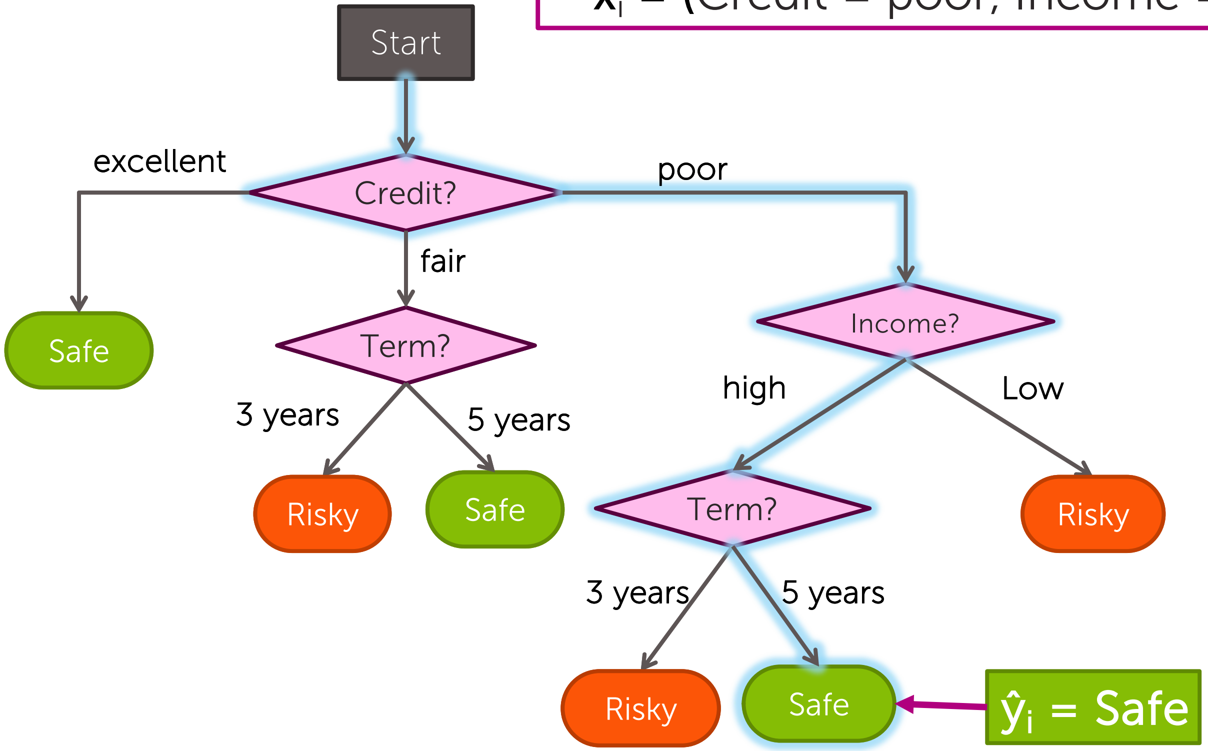
$\hat{y}_i = -1$

This module … decision trees



Scoring a loan application

$\mathbf{x}_i$ = (Credit = poor, Income = high, Term = 5 years)



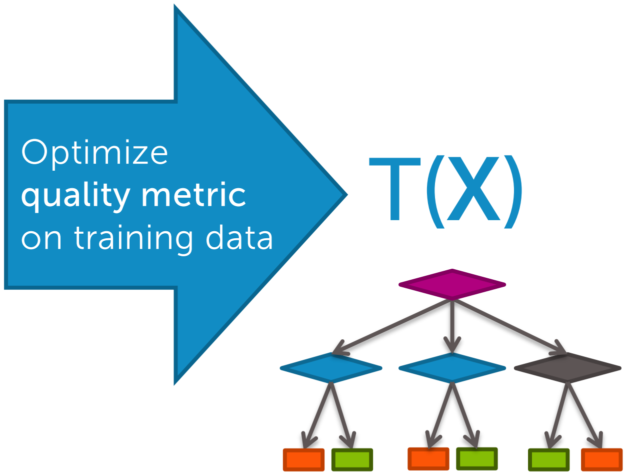
$\hat{y}_i$ = Safe

Decision tree learning task

Decision tree learning problem

Training data: N observations ($\mathbf{x}_i$,y_i)

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe



Optimize **quality metric** on training data

T(X)

Quality metric: Classification error

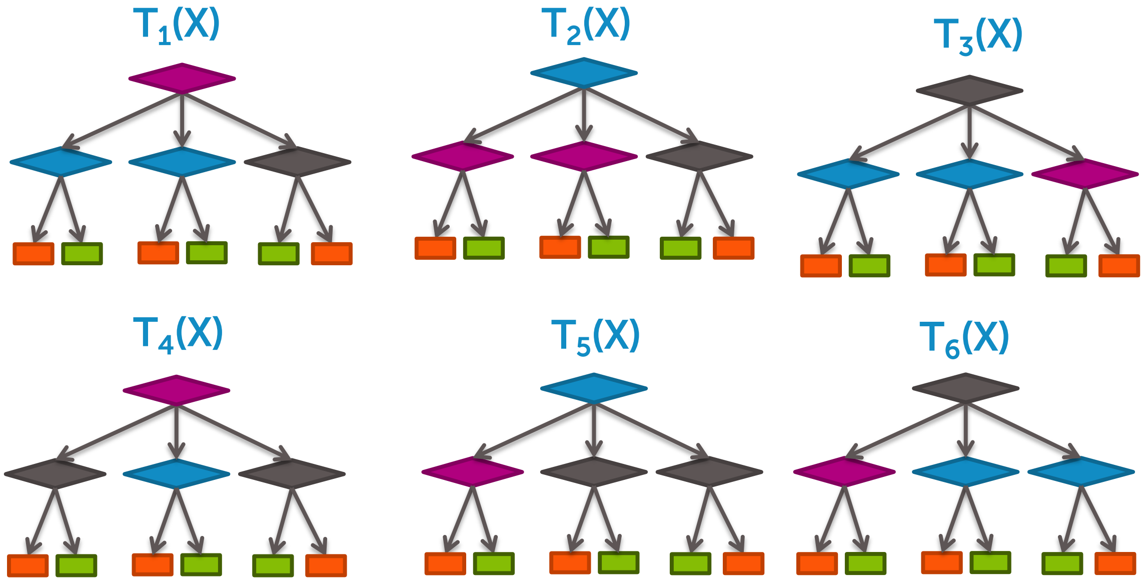
- Error measures fraction of mistakes

$$\boxed{\text{Error} = \frac{\text{\# incorrect predictions}}{\text{\# examples}}}$$

  - Best possible value : 0.0
  - Worst possible value: 1.0

How do we find the best tree?

Exponentially large number of possible trees makes decision tree learning hard!



Learning the smallest decision tree is an *NP-hard problem* [Hyafil & Rivest '76]

Greedy decision tree learning

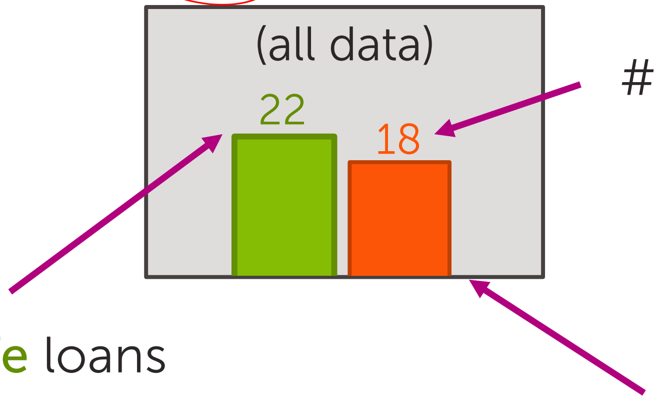
Our training data table

Assume N = 40, 3 features

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe

Start with all the data

Loan status: Safe Risky



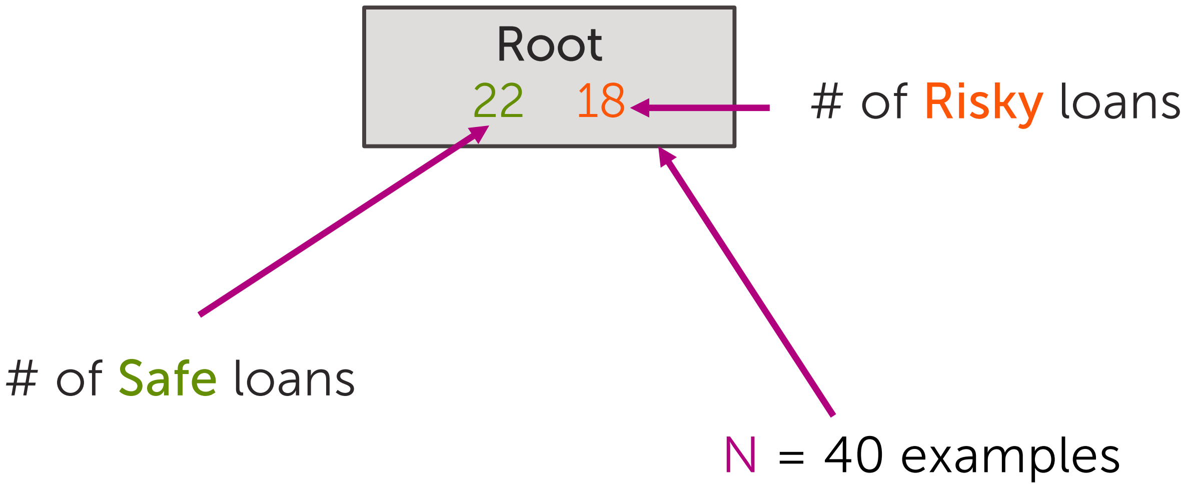
# of Risky loans

# of Safe loans

N = 40 examples

Compact visual notation: Root node

Loan status: Safe Risky



Root

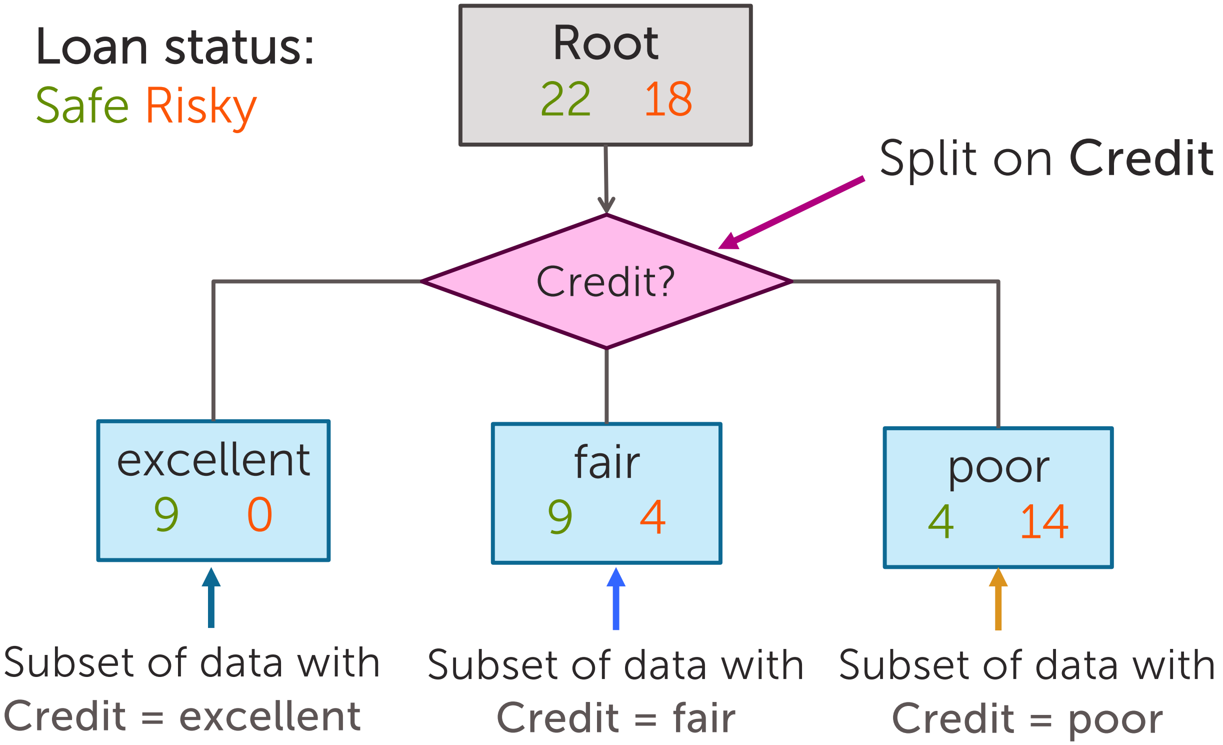
22 18

# of Risky loans

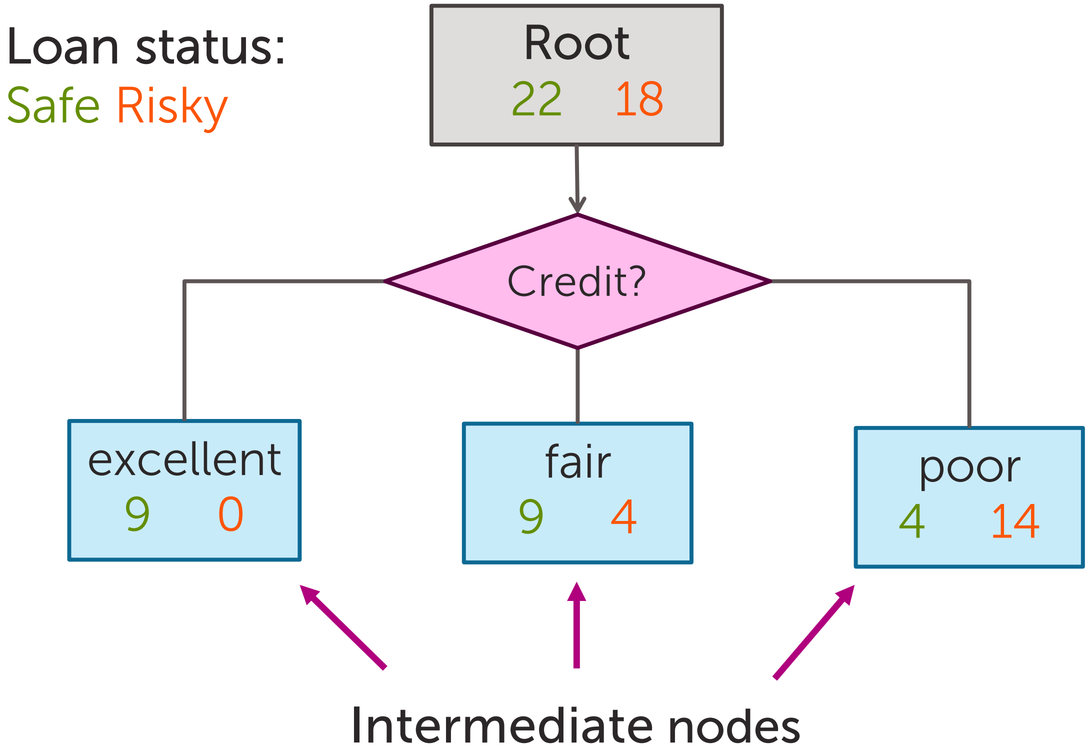
# of Safe loans

N = 40 examples

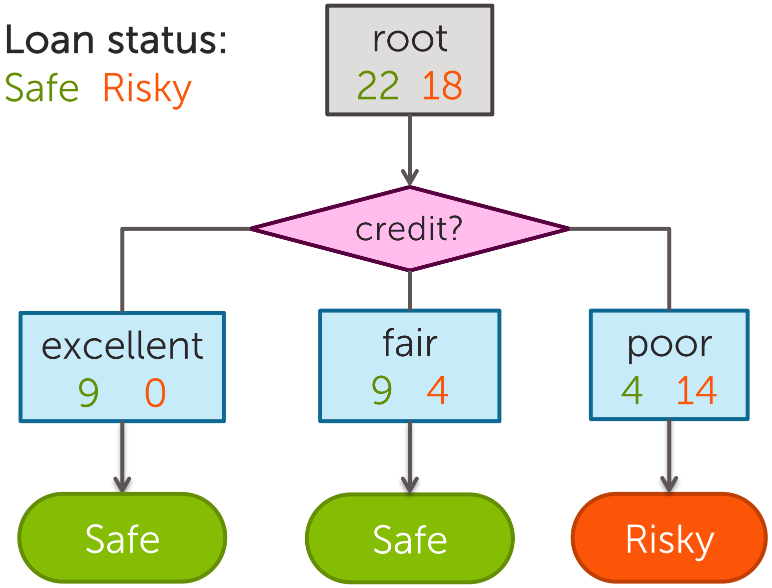
Decision stump: Single level tree


Loan status:
Safe Risky
Root
22 18
Split on **Credit**
Credit?
excellent
9 0
fair
9 4
poor
4 14
Subset of data with Credit = excellent
Subset of data with Credit = fair
Subset of data with Credit = poor

Visual notation: Intermediate nodes



Making predictions with a decision stump

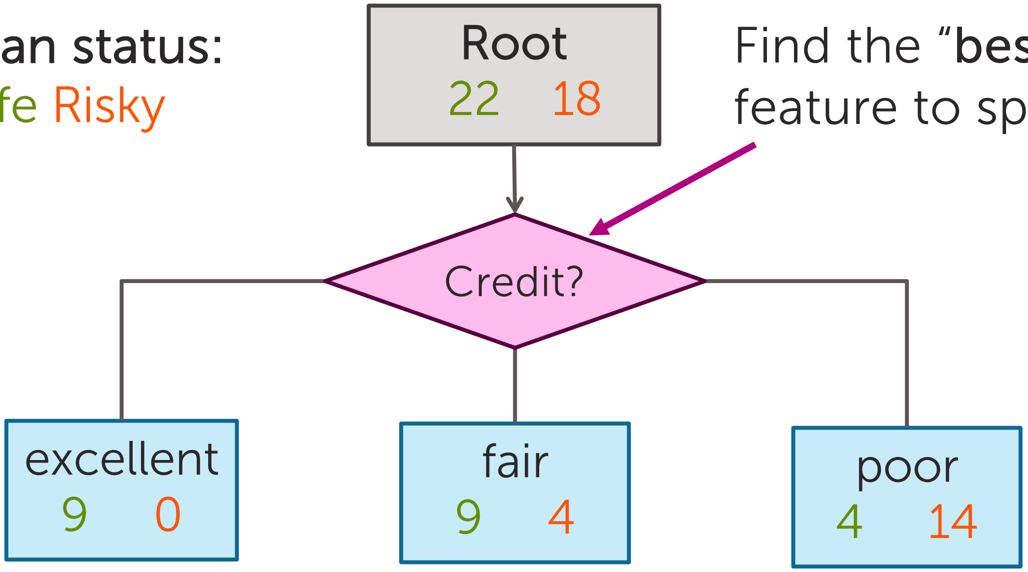


For each intermediate node, set $\hat{y}$ = **majority value**

Selecting best feature to split on

How do we learn a decision stump?

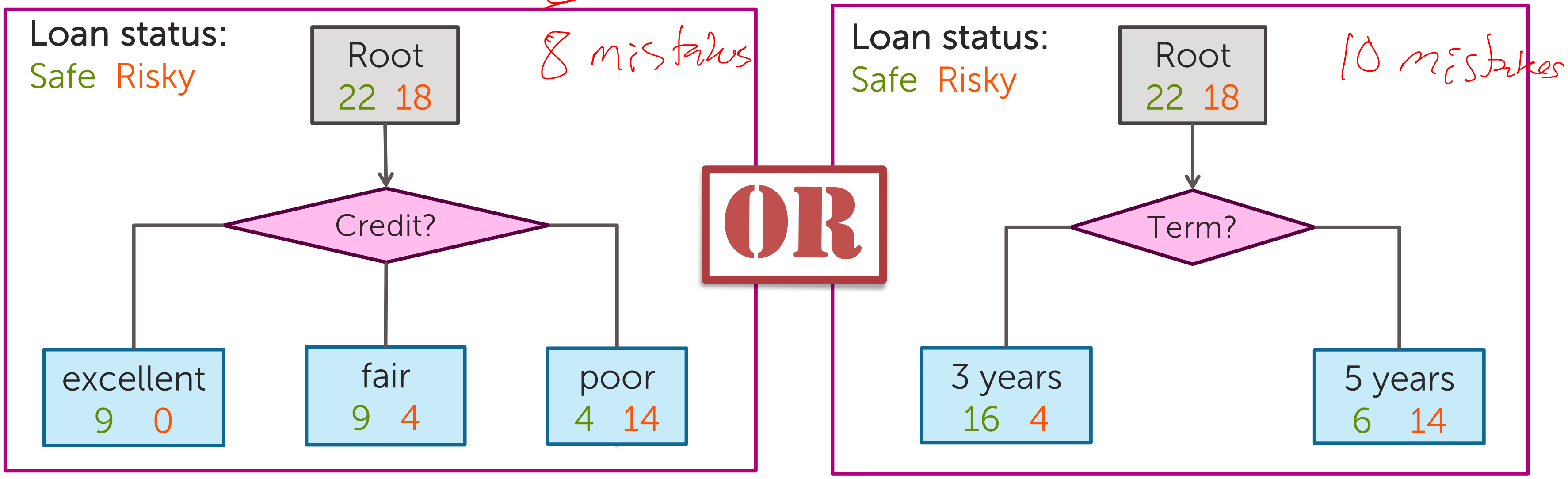
Loan status:
Safe Risky



Find the "**best**" feature to split on!

How do we select the best feature?

which one makes fewer mistakes?



Choice 1: Split on **Credit**

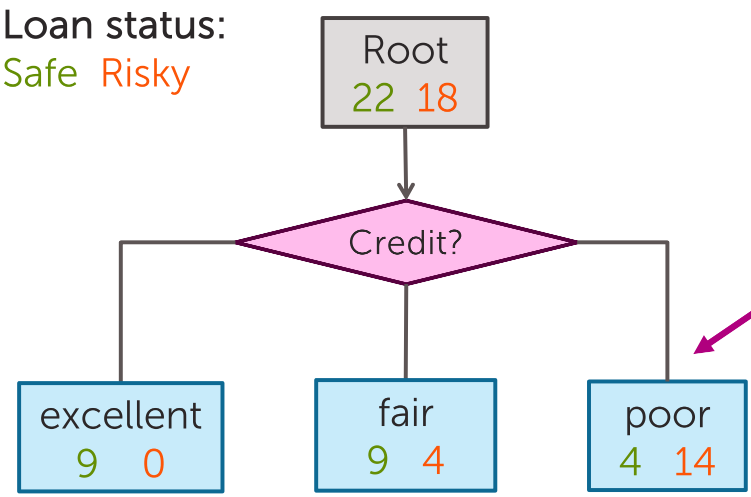
8 mistakes

OR

Choice 2: Split on **Term**

10 mistakes

How do we measure effectiveness of a split?

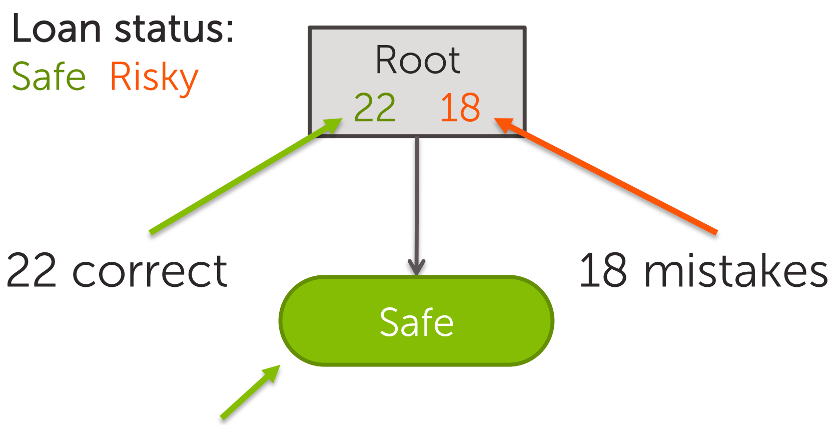


Idea: Calculate classification error of this decision stump

$$\text{Error} = \frac{\#\text{ mistakes}}{\#\text{ data points}}$$

Calculating classification error

- **Step 1:** ŷ = class of majority of data in node
- **Step 2:** Calculate classification error of predicting ŷ for this data



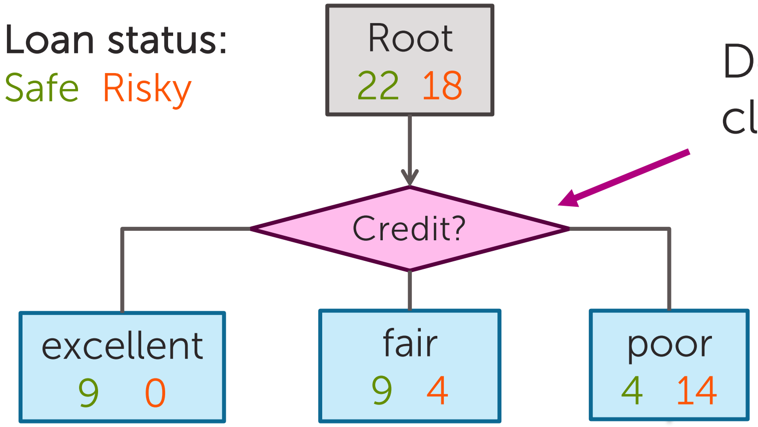
ŷ = majority class

$$\text{Error} = \frac{18}{40} = 0.45$$

Tree	Classification error
(root)	0.45

Choice 1: Split on Credit history?

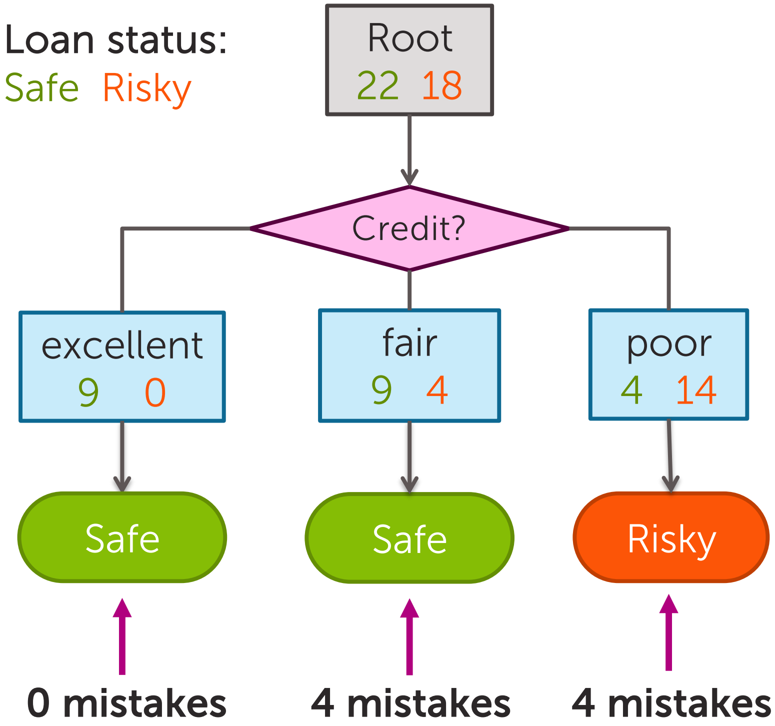
Choice 1: Split on **Credit**



Does a split on Credit reduce classification error below 0.45?

Split on Credit: Classification error

Choice 1: Split on Credit



Loan status:
Safe Risky

Root
22 18

Credit?

excellent
9 0

fair
9 4

poor
4 14

Safe

Safe

Risky

0 mistakes

4 mistakes

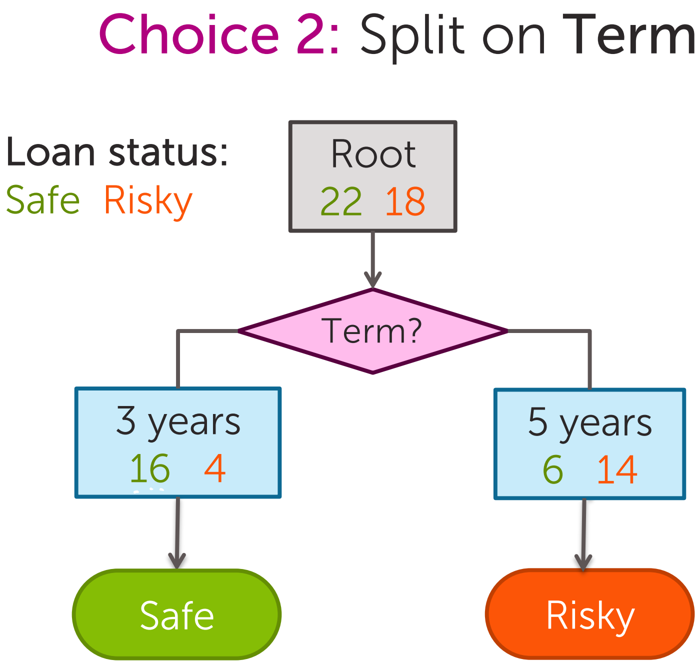
4 mistakes

Error = $\frac{8}{40}$

= 0.2

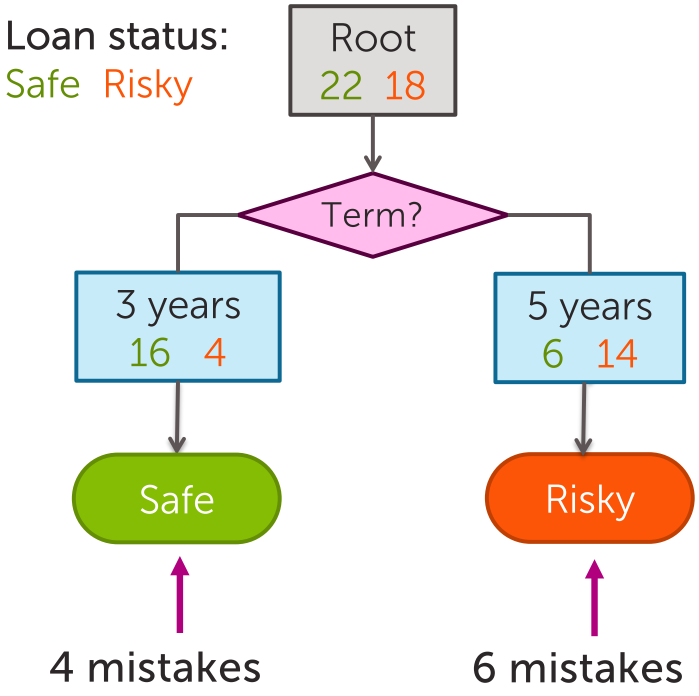
Tree	Classification error
(root)	0.45
Split on **credit**	0.2

Choice 2: Split on Term?



Evaluating the split on Term

Choice 2: Split on **Term**



Loan status:
Safe Risky

Root
22 18

Term?

3 years
16 4

5 years
6 14

Safe

Risky

4 mistakes

6 mistakes

$$\text{Error} = \frac{10}{40}$$

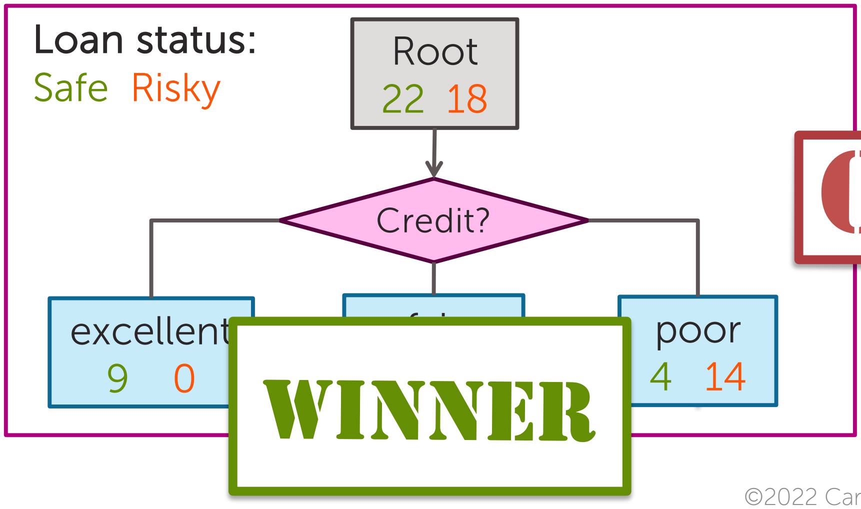
$$= 0.25$$

Tree	Classification error
(root)	0.45
Split on **credit**	0.2
Split on **term**	0.25

Choice 1 vs Choice 2: Comparing split on Credit vs Term

Tree	Classification error
(root)	0.45
split on **credit**	0.2
split on **loan term**	0.25

Choice 1: Split on **Credit**


Loan status:
Safe Risky

Root
22 18

Credit?

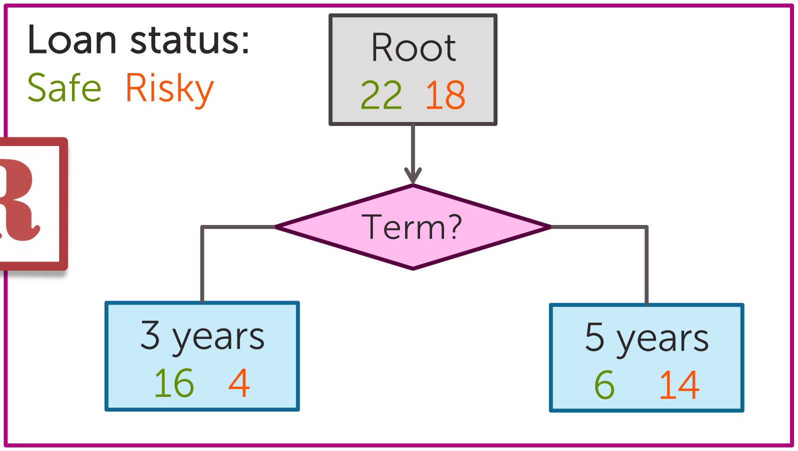
excellent
9 0

poor
4 14

WINNER

OR

Choice 2: Split on **Term**


Loan status:
Safe Risky

Root
22 18

Term?

3 years
16 4

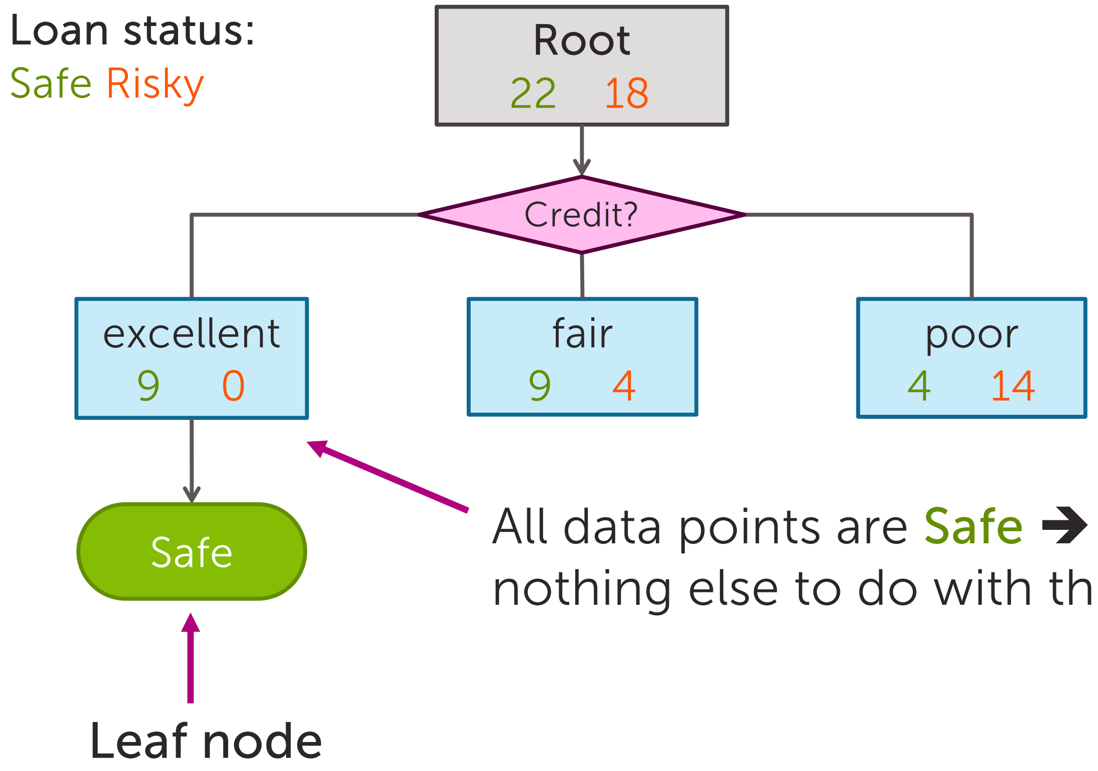
5 years
6 14

Feature split selection algorithm

- Given a subset of data M (a node in a tree)
- For each feature $h_i(x)$:
  1. Split data of M according to feature $h_i(x)$
  2. Compute classification error of split
- Chose feature $h^*(x)$ with lowest classification error

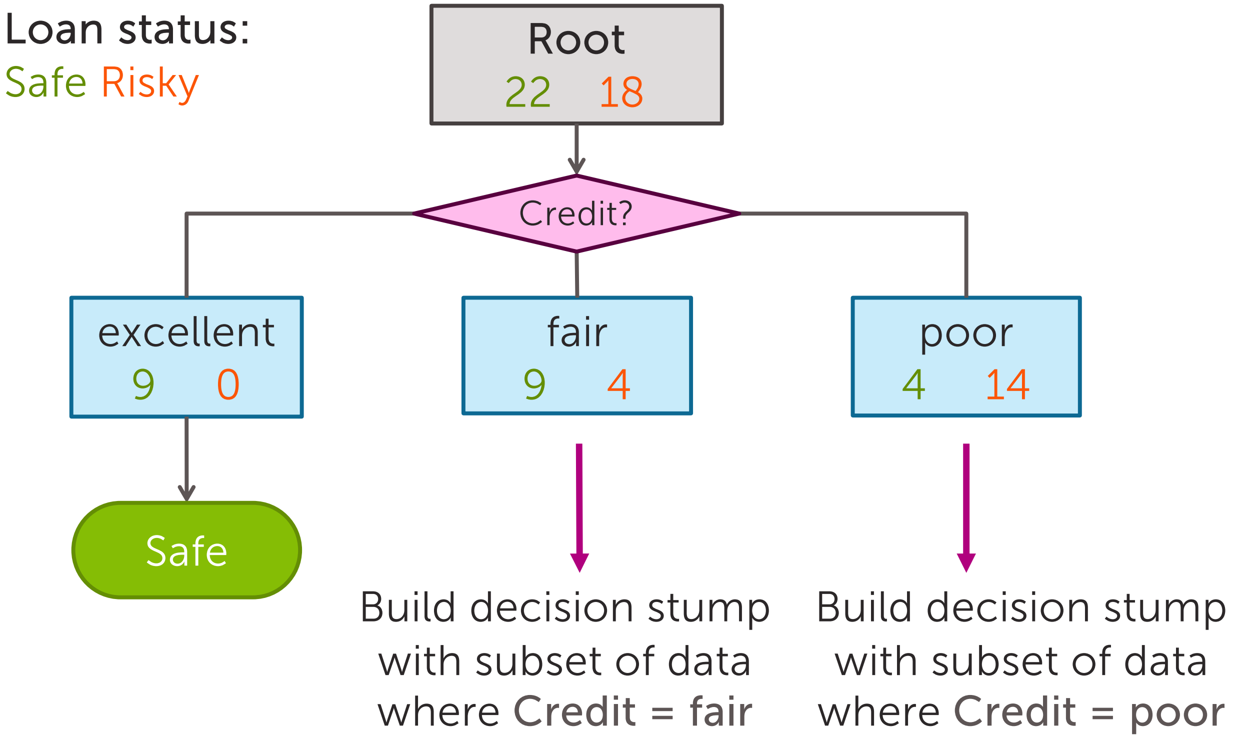
Recursion & Stopping conditions

We’ve learned a decision stump, what next?



All data points are **Safe** ➔
nothing else to do with this subset of data

Tree learning = Recursive stump learning

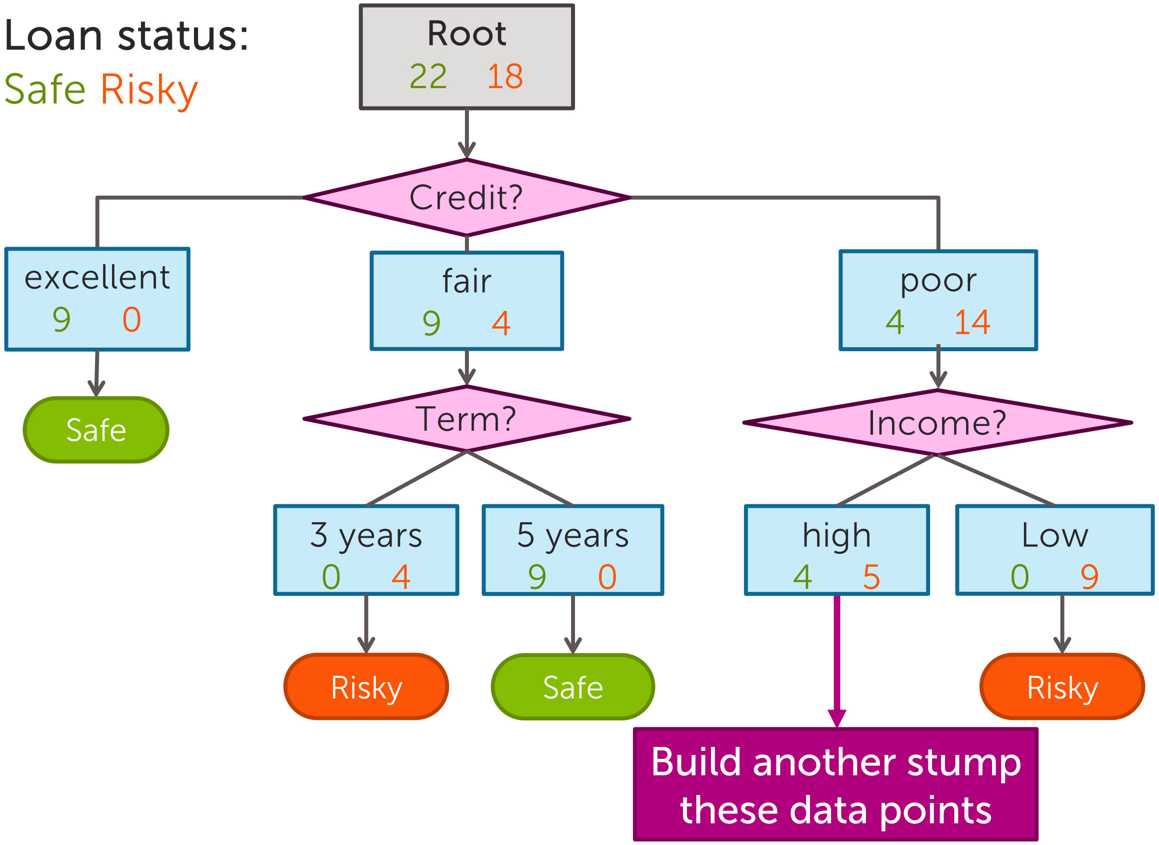


Build decision stump with subset of data where Credit = fair

Build decision stump with subset of data where Credit = poor

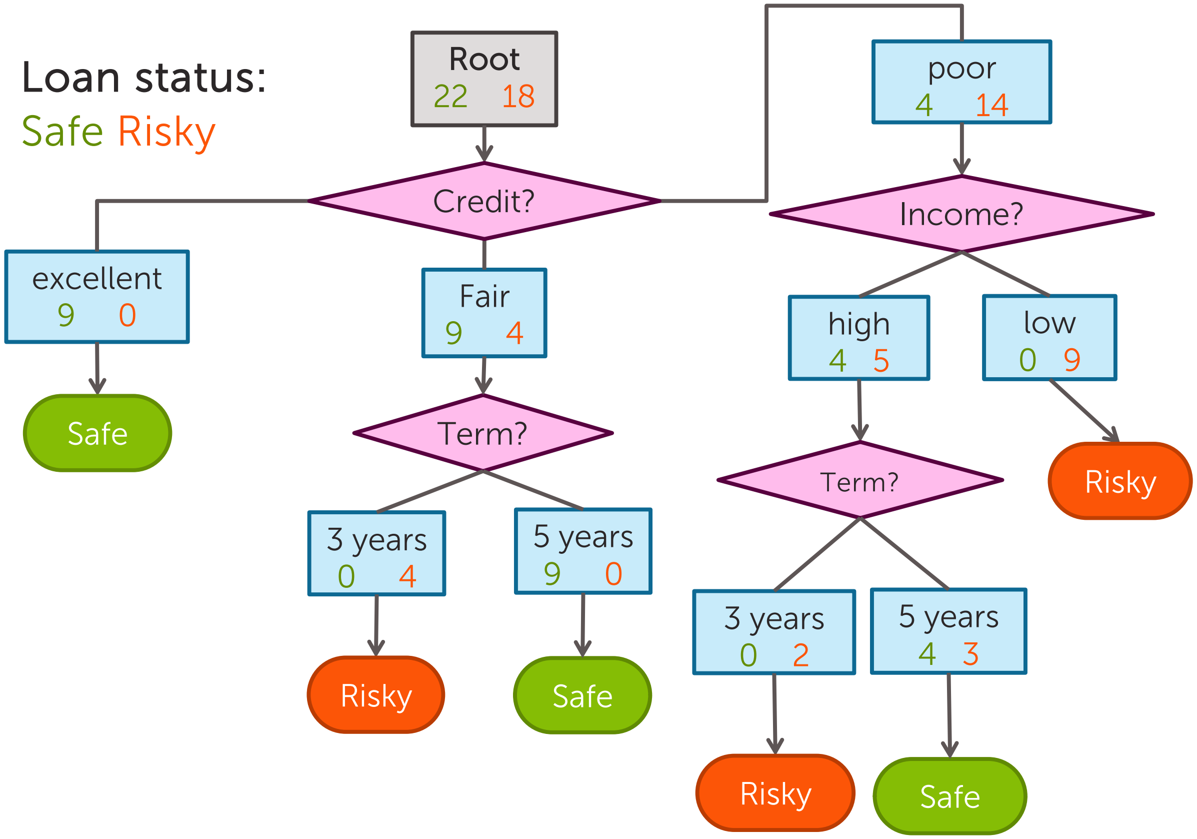
Second level

Loan status:
Safe Risky

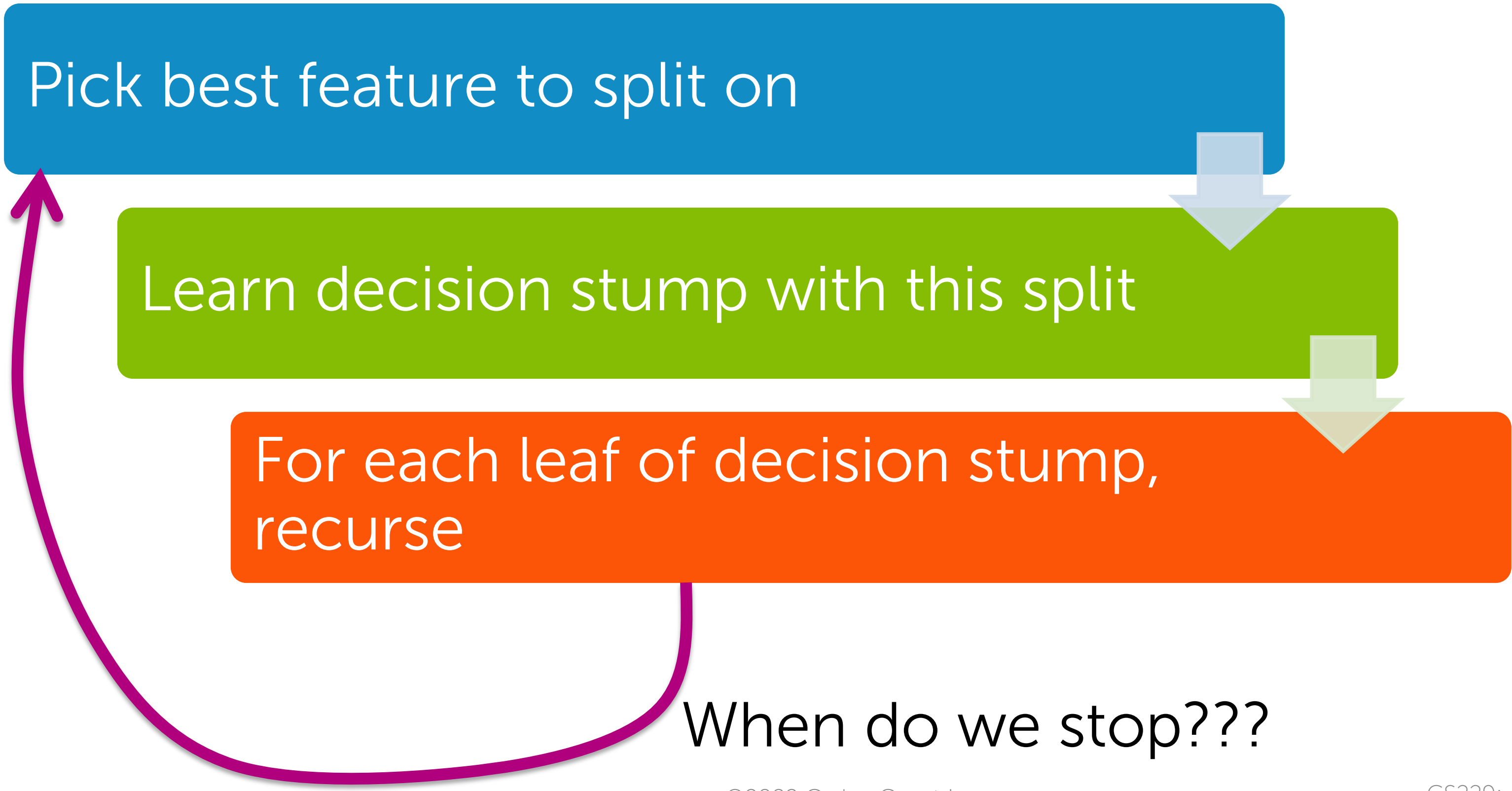


Final decision tree

Loan status:
Safe Risky



Simple greedy decision tree learning



Pick best feature to split on

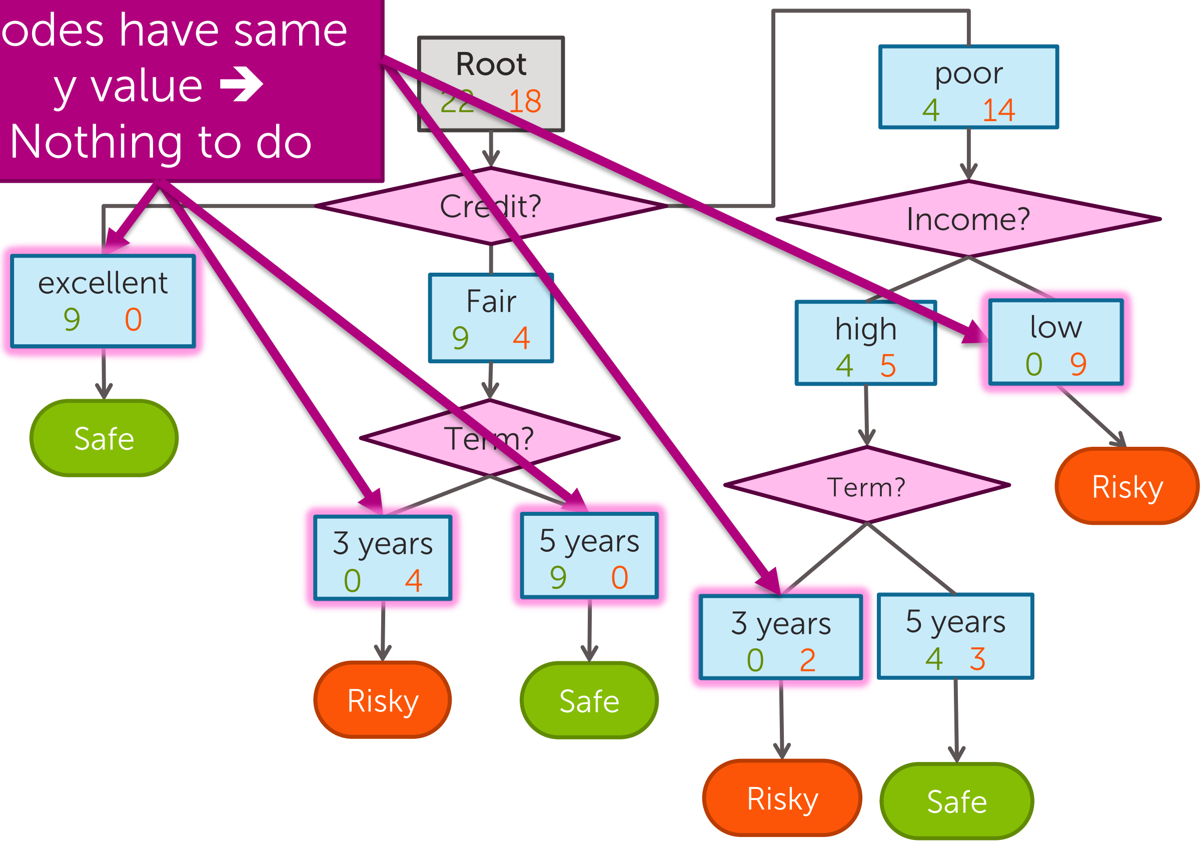
Learn decision stump with this split

For each leaf of decision stump, recurse

When do we stop???

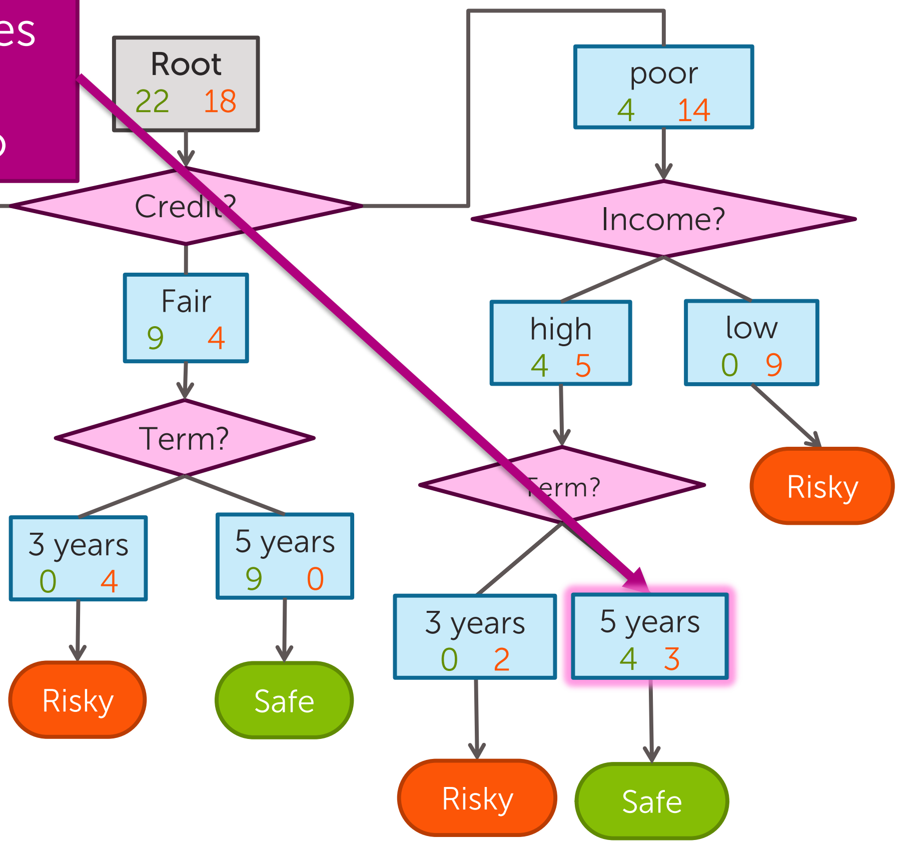
Stopping condition 1: All data agrees on y

All data in these nodes have same y value ➔ Nothing to do

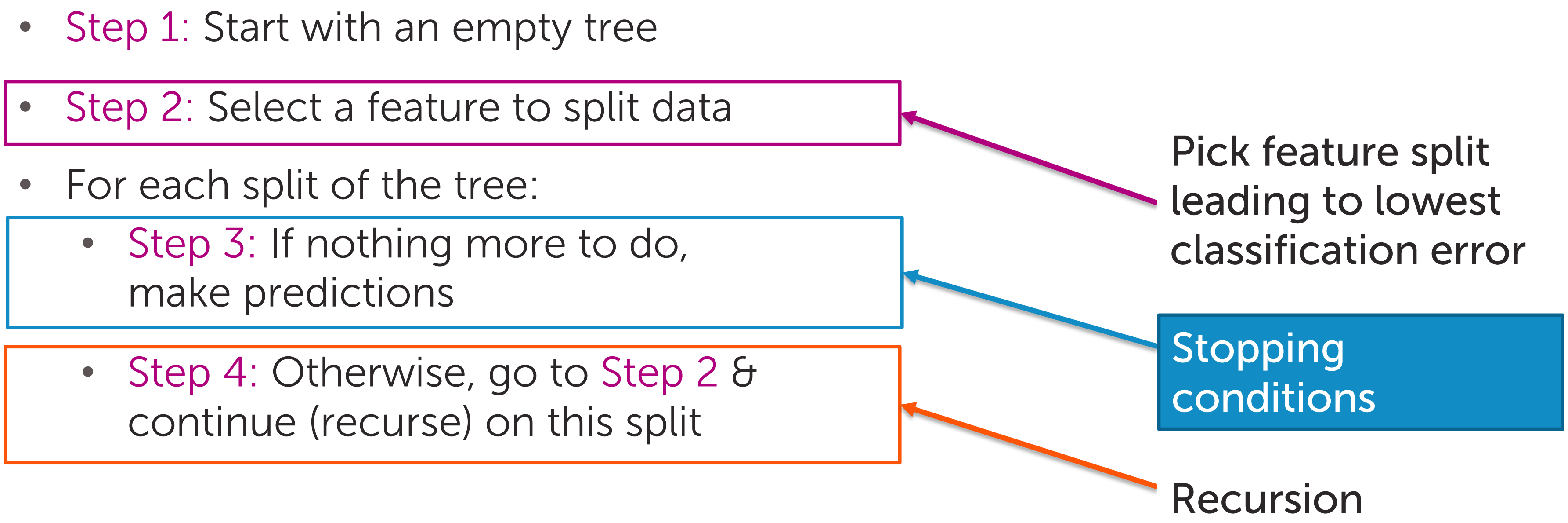


Stopping condition 2: Already split on all features

Already split on all possible features
➔
Nothing to do



Greedy decision tree learning



- Step 1: Start with an empty tree
- Step 2: Select a feature to split data
- For each split of the tree:
  - Step 3: If nothing more to do, make predictions
  - Step 4: Otherwise, go to Step 2 & continue (recurse) on this split

Pick feature split leading to lowest classification error

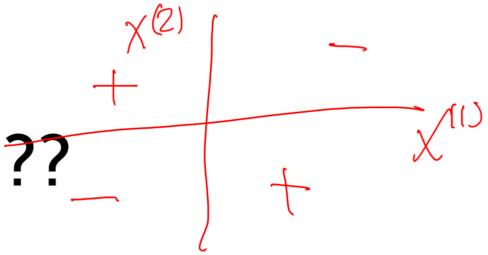
Stopping conditions

Recursion

Is this a good idea?

Proposed stopping condition 3:
Stop if no split reduces the classification error

Stopping condition 3:
Don't stop if error doesn't decrease???



$y = \mathbf{x}[1]$ xor $\mathbf{x}[2]$

x[1]	**x**[2]	y
False	False	False
False	True	True
True	False	True
True	True	False

y values
True False

Root
2 2

Error = $\frac{2}{4}$

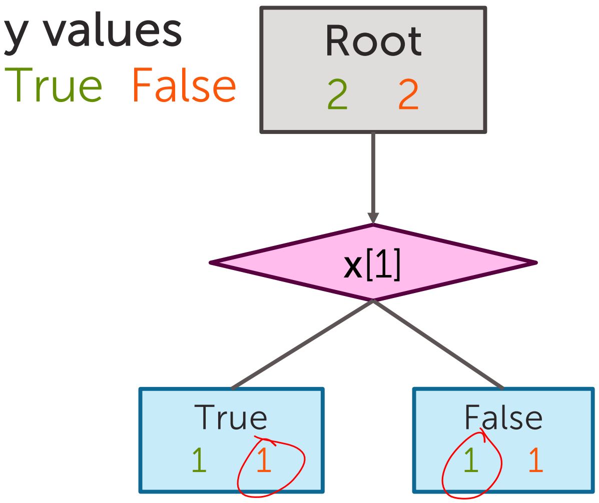
= 0.5

Tree	Classification error
(root)	0.5

Consider split on x[1]

y = **x**[1] xor **x**[2]

x[1]	**x**[2]	y
False	False	False
False	True	True
True	False	True
True	True	False



Error = $\frac{2}{4}$

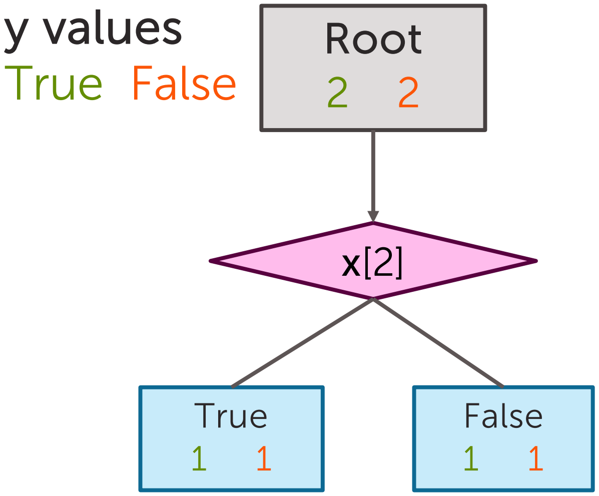
= 0.5

Tree	Classification error
(root)	0.5
Split on **x**[1]	0.5

Consider split on x[2]

y = **x**[1] xor **x**[2]

x[1]	**x**[2]	y
False	False	False
False	True	True
True	False	True
True	True	False



$$\text{Error} = \frac{1+1}{2+2}$$

$$= 0.5$$

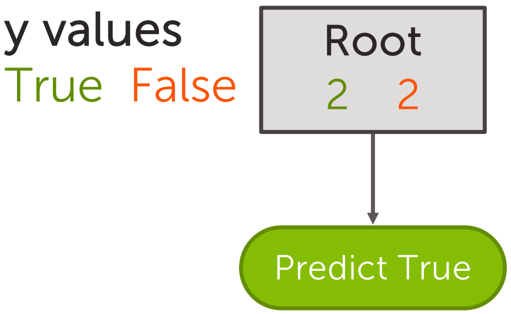
Neither features improve training error… Stop now???

Tree	Classification error
(root)	0.5
Split on **x**[1]	0.5
Split on **x**[2]	0.5

Final tree with stopping condition 3

$y = \mathbf{x}[1] \text{ xor } \mathbf{x}[2]$

x[1]	**x**[2]	y
False	False	False
False	True	True
True	False	True
True	True	False


y values
True False
Root
2 2
Predict True

Tree	Classification error
with stopping condition 3	0.5

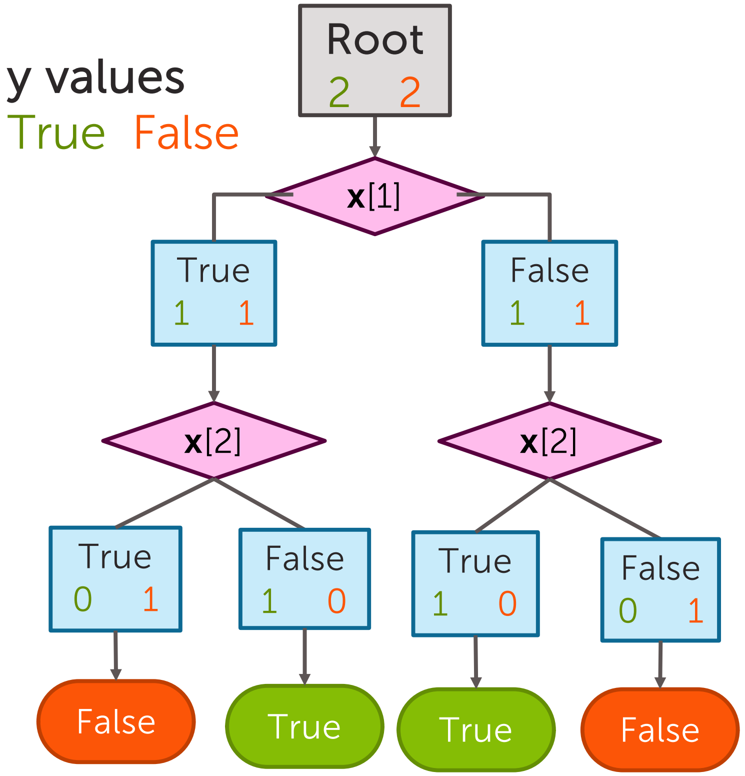
Without stopping condition 3

Condition 3 (stopping when training error doesn't' improve) is **not** recommended!

y = **x**[1] xor **x**[2]

x[1]	**x**[2]	y
False	False	False
False	True	True
True	False	True
True	True	False

Tree	Classification error
with stopping condition 3	0.5
without stopping condition 3	0



Decision tree learning: *Real valued features*

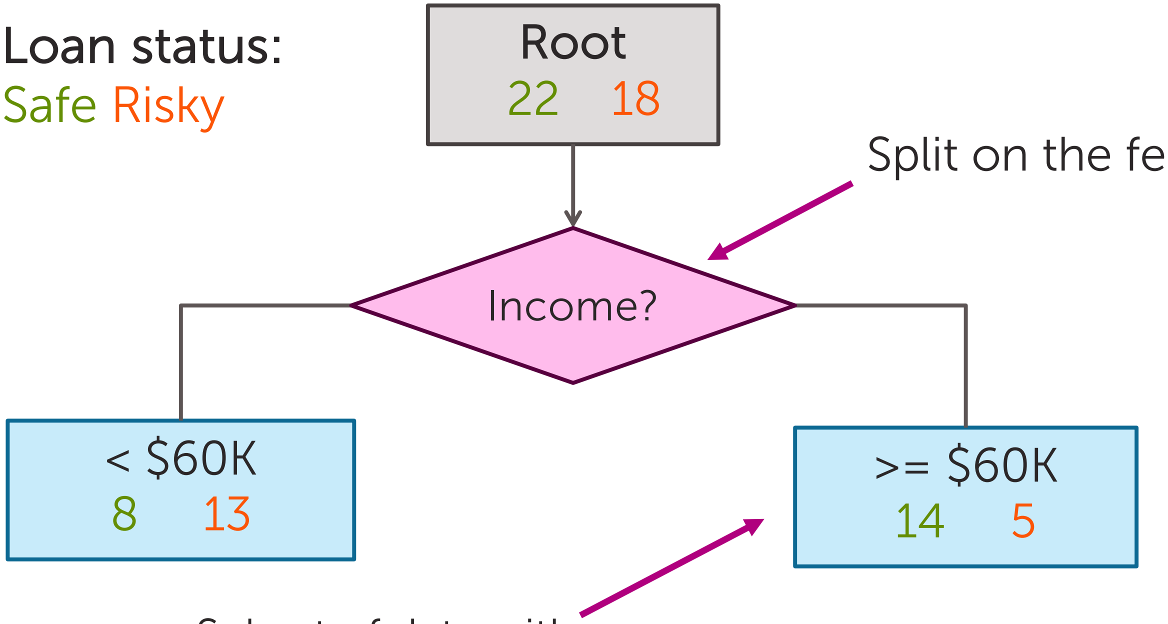
How do we use real values inputs?

Income	Credit	Term	y
$105 K	excellent	3 yrs	Safe
$112 K	good	5 yrs	Risky
$73 K	fair	3 yrs	Safe
$69 K	excellent	5 yrs	Safe
$217 K	excellent	3 yrs	Risky
$120 K	good	5 yrs	Safe
$64 K	fair	3 yrs	Risky
$340 K	excellent	5 yrs	Safe
$60 K	good	3 yrs	Risky

Threshold split

turn continuous var into binary

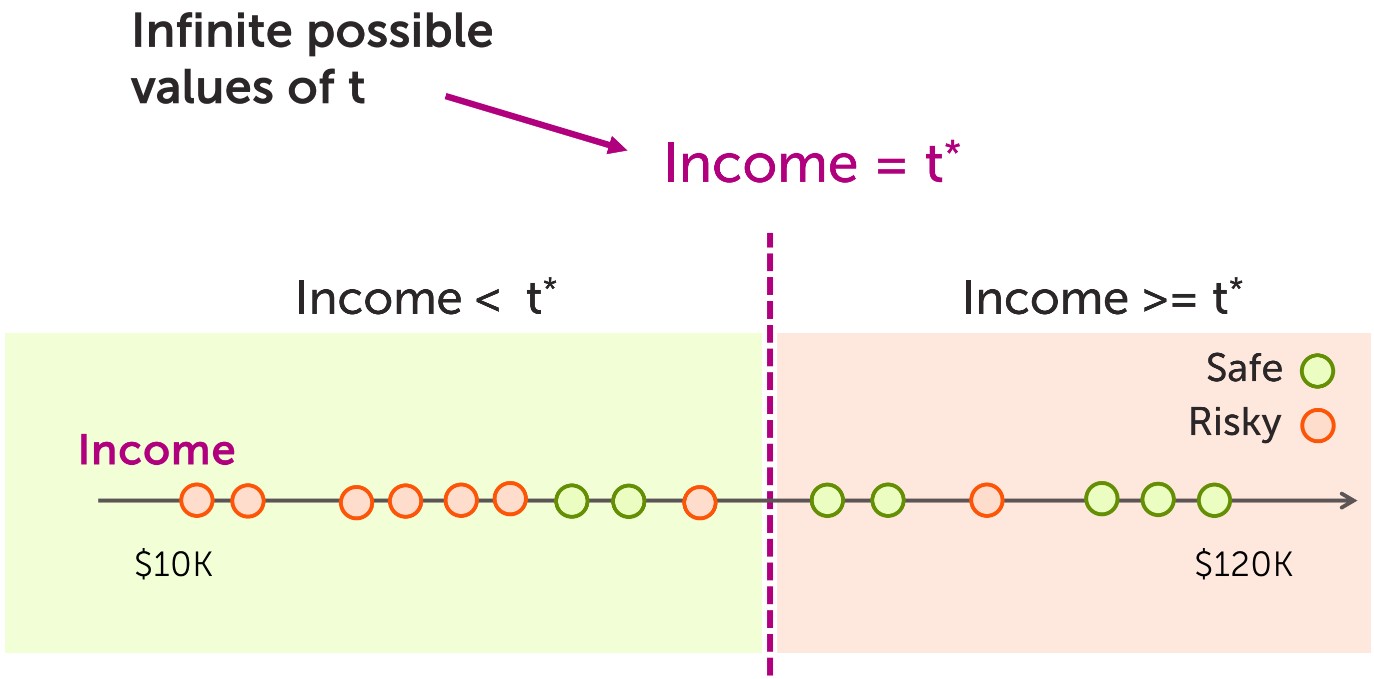
Loan status:
Safe Risky



Split on the feature **Income**

Subset of data with
Income >= $60K

Finding the best threshold split


Infinite possible values of t

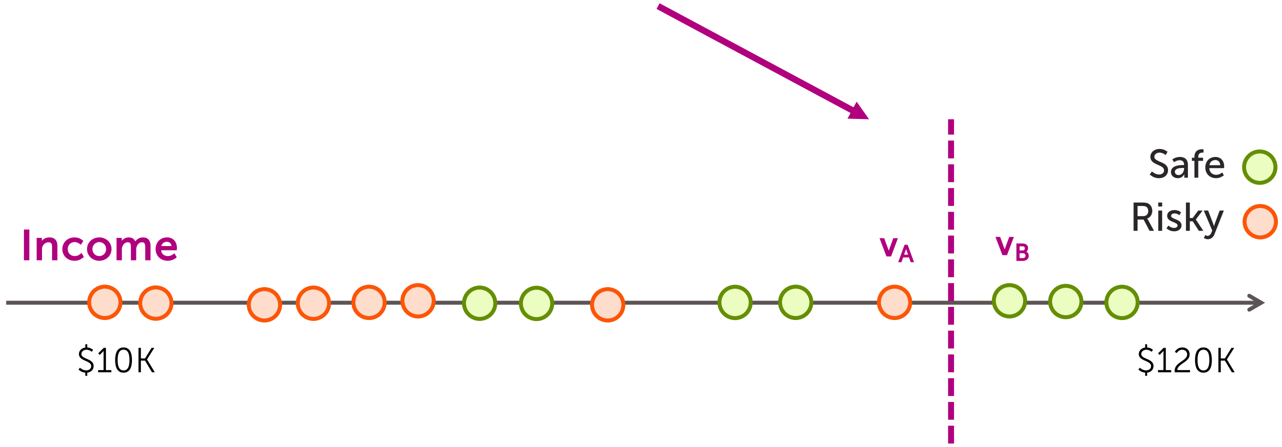
$\text{Income} = t^*$

$\text{Income} < t^*$

$\text{Income} >= t^*$

Consider a threshold between points

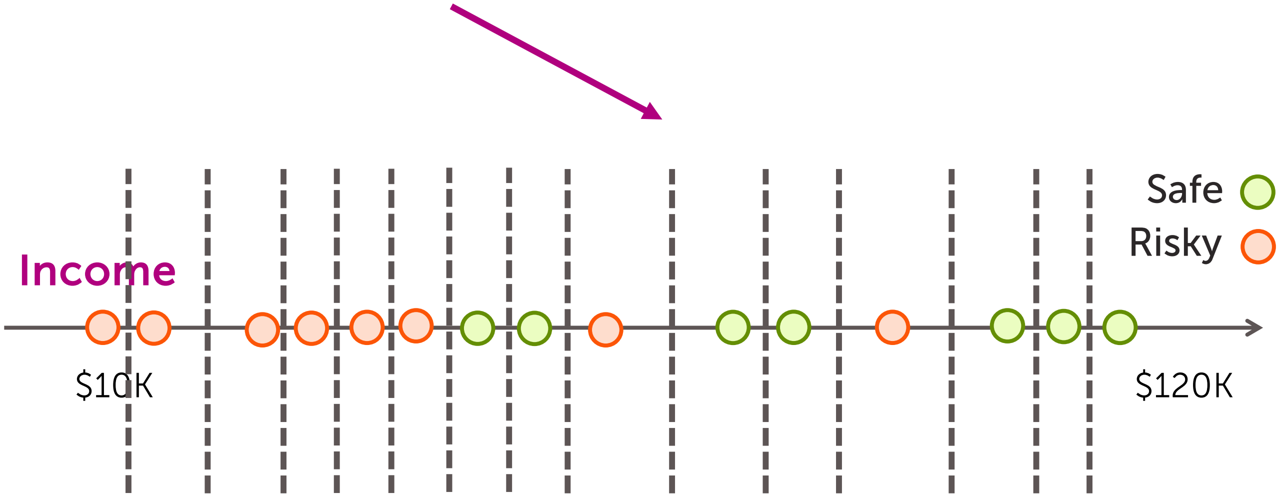
Same classification error for any threshold split between v_A and v_B



Only need to consider mid-points

Sorted data:

Finite number of splits to consider

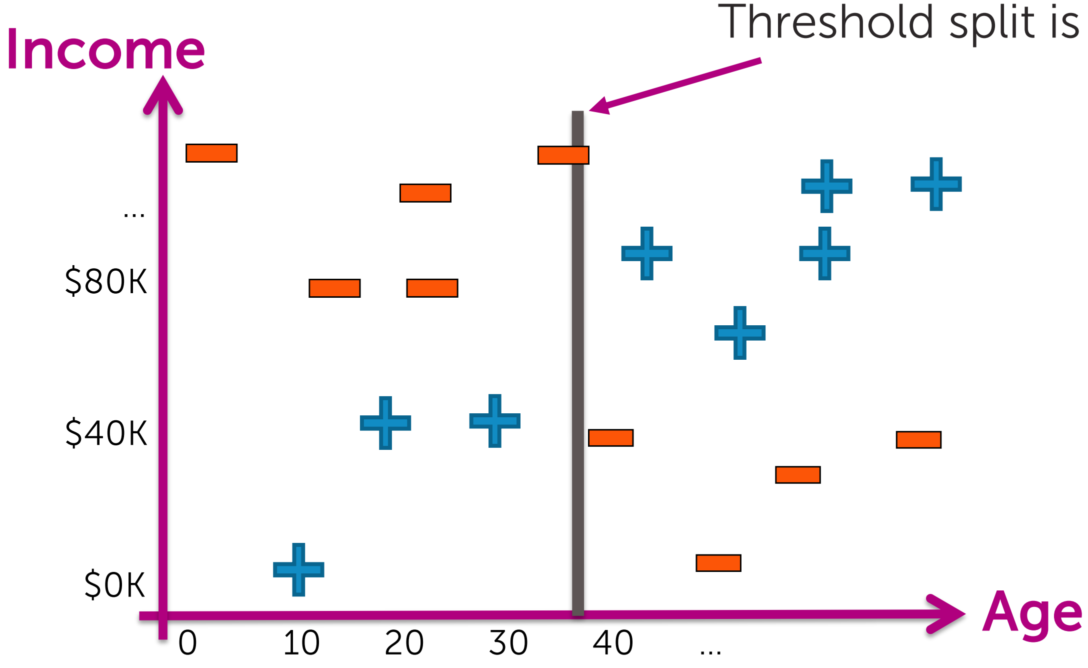


Threshold split selection algorithm

- Step 1: Sort the values of a feature $h_j(\mathbf{x})$:

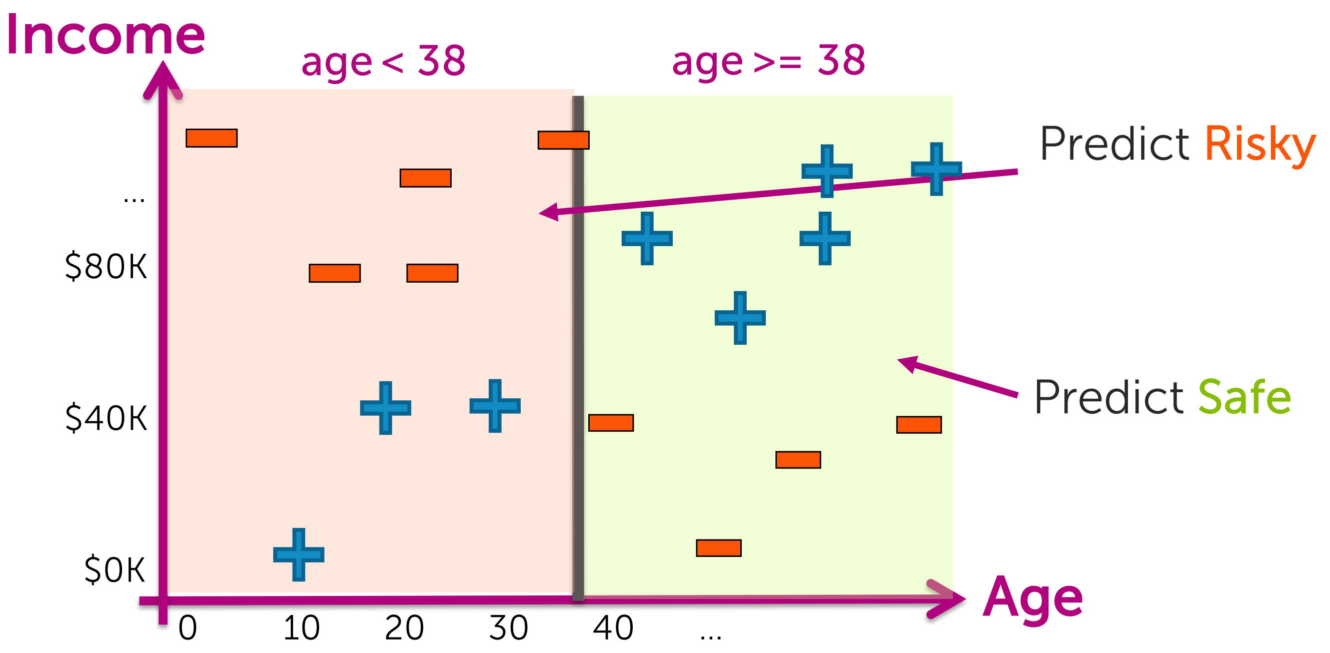
  Let $\{v_1, v_2, v_3, \ldots v_N\}$ denote sorted values
- Step 2:
  - For i = 1 … N-1
    - Consider split $t_i = (v_i + v_{i+1}) / 2$
    - Compute classification error for treshold split $h_j(\mathbf{x}) >= \mathbf{t}_i$
  - Chose the $\mathbf{t}^*$ with the lowest classification error

Visualizing the threshold split

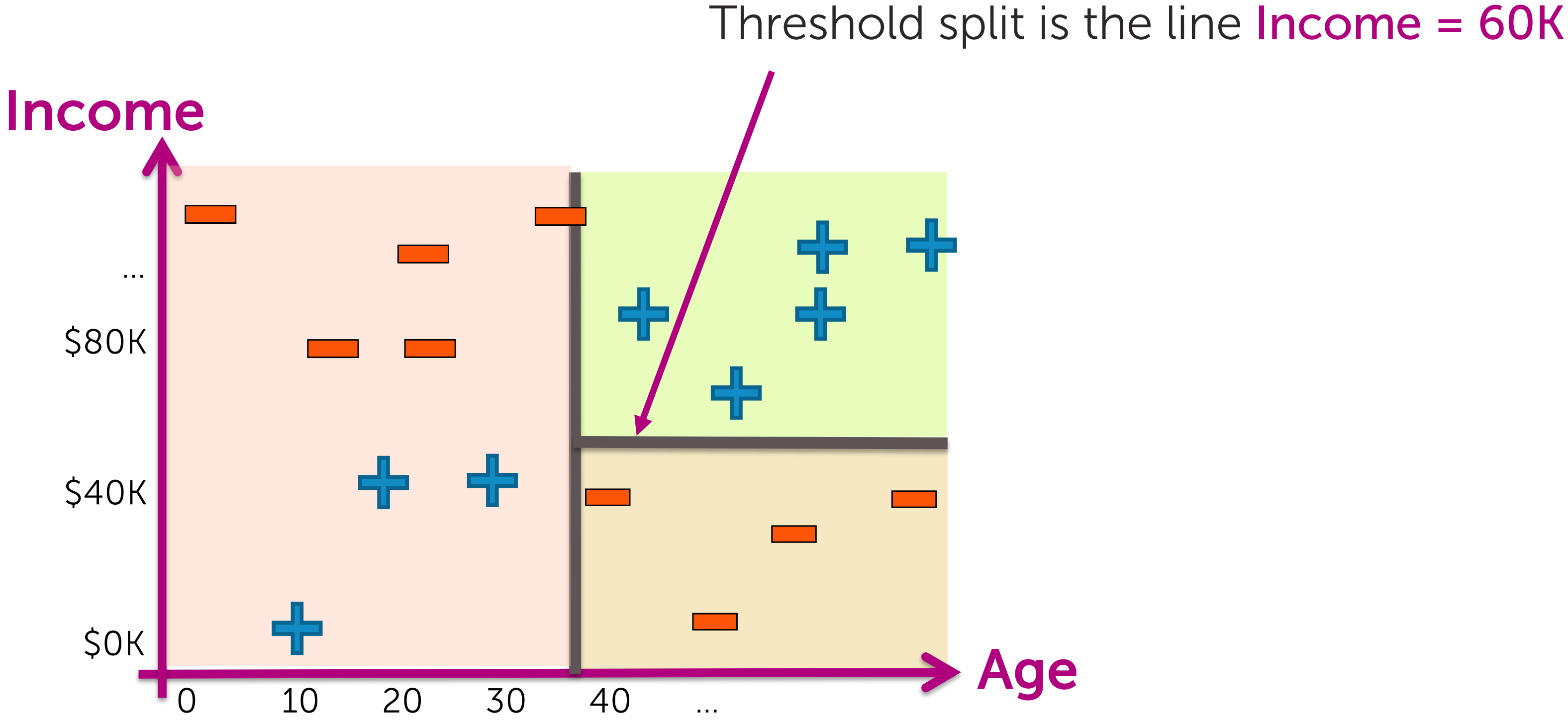


Threshold split is the line Age = 38

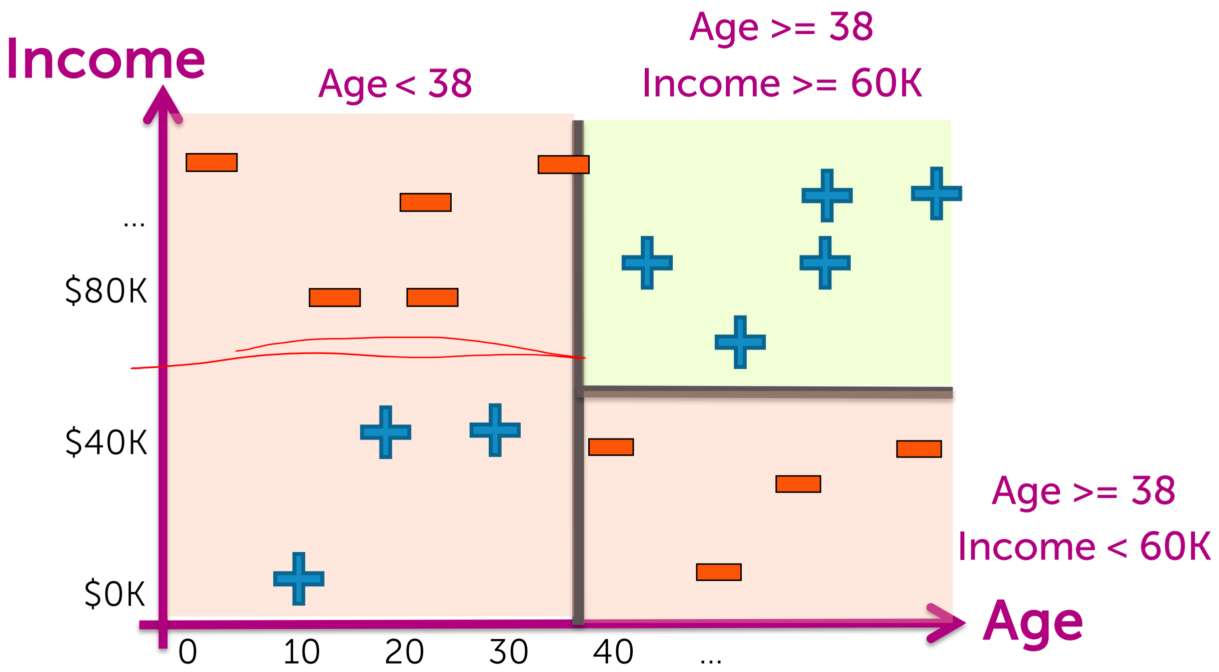
Split on Age >= 38



Depth 2: Split on Income >= $60K



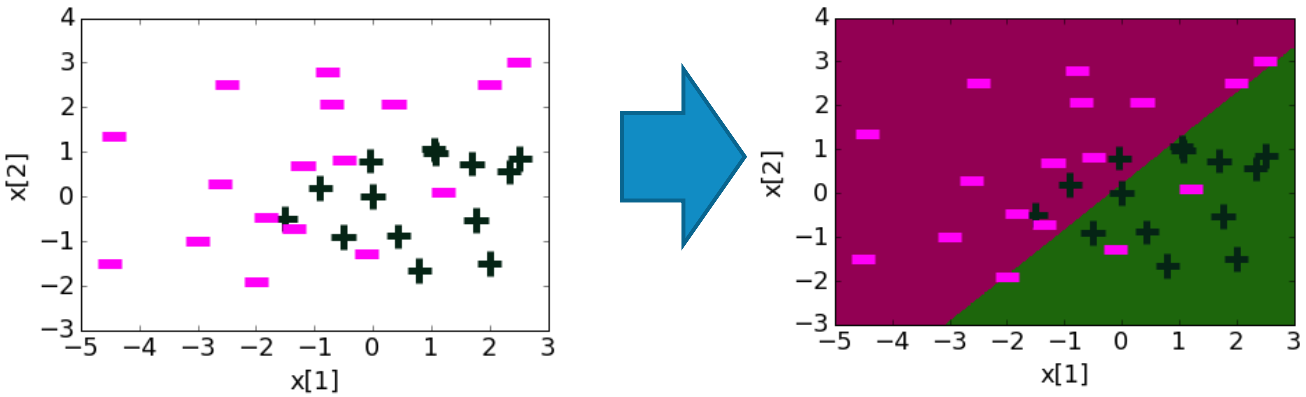
Each split partitions the 2-D space



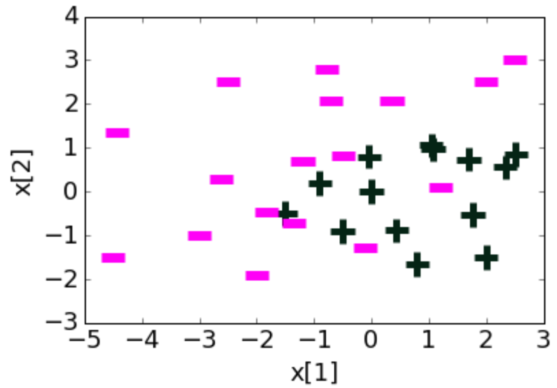
Decision trees vs logistic regression: *Example*

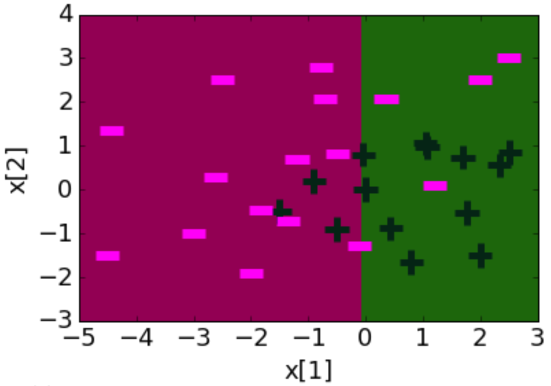
Logistic regression

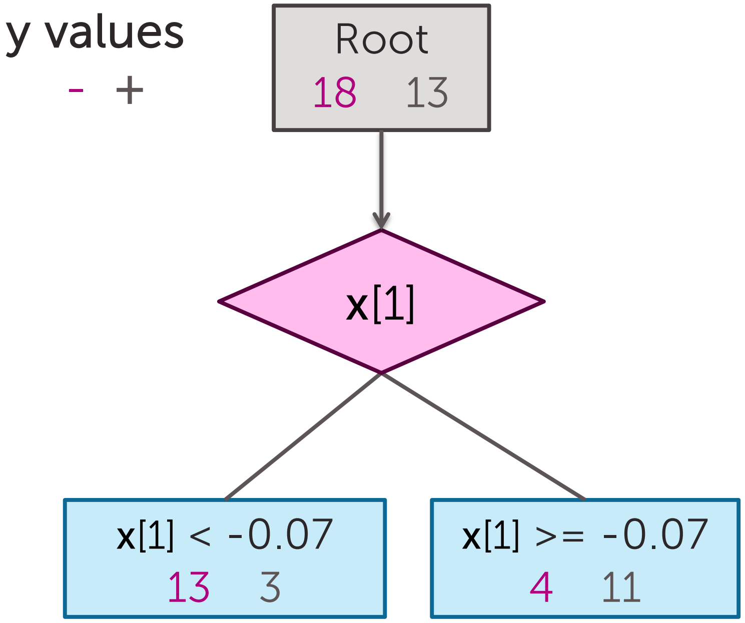
Feature	Value	Weight Learned
$h_0(\mathbf{x})$	1	0.22
$h_1(\mathbf{x})$	x[1]	1.12
$h_2(\mathbf{x})$	x[2]	-1.07



Depth 1: Split on x[1]







y values

- +

Root

18 13

x[1]

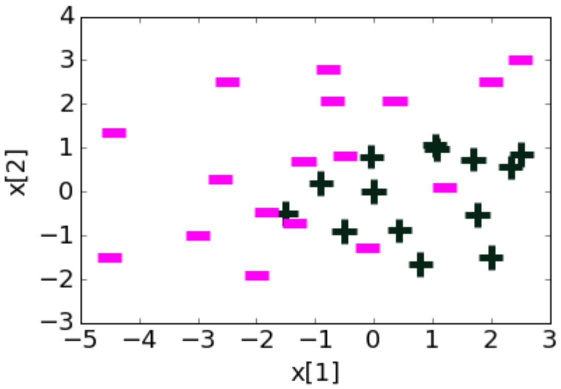
x[1] < -0.07

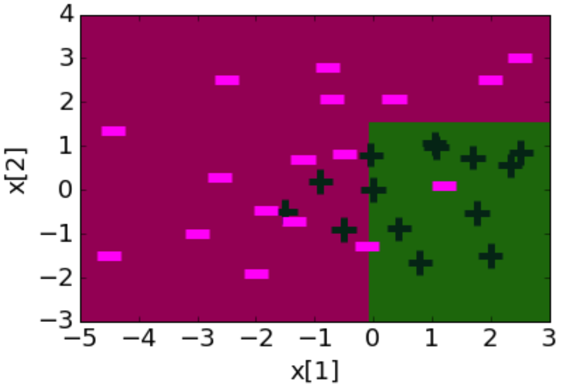
13 3

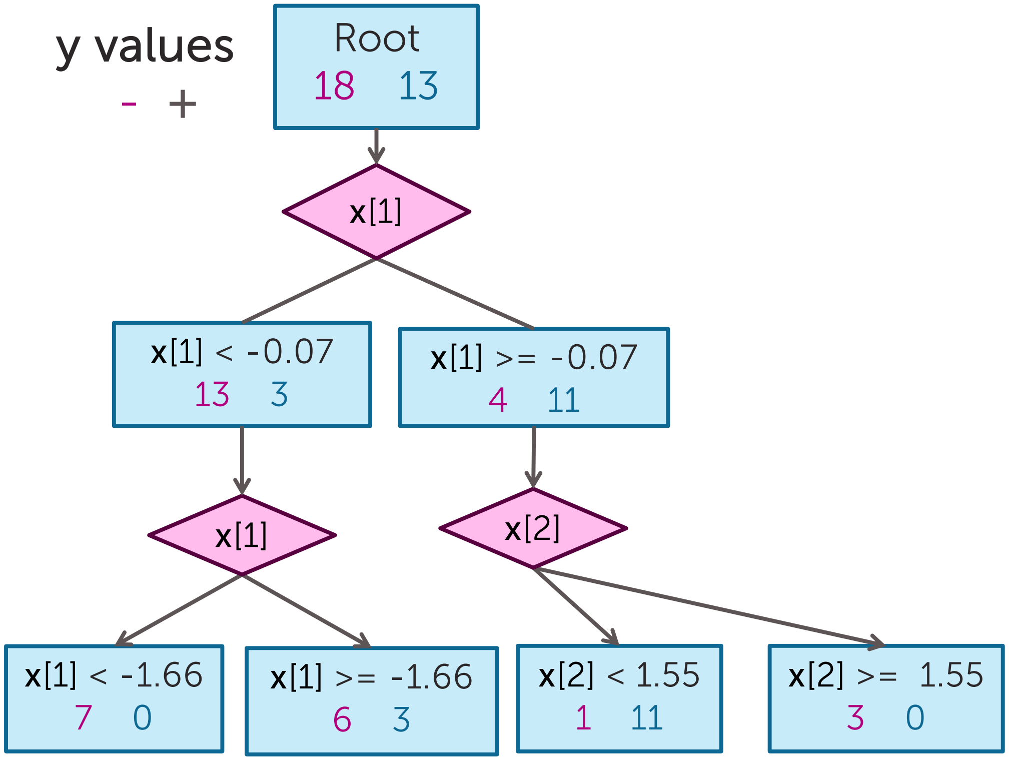
x[1] >= -0.07

4 11

Depth 2







y values

- +

Root

18 13

x[1]

x[1] < -0.07

13 3

x[1] >= -0.07

4 11

x[1]

x[2]

x[1] < -1.66

7 0

x[1] >= -1.66

6 3

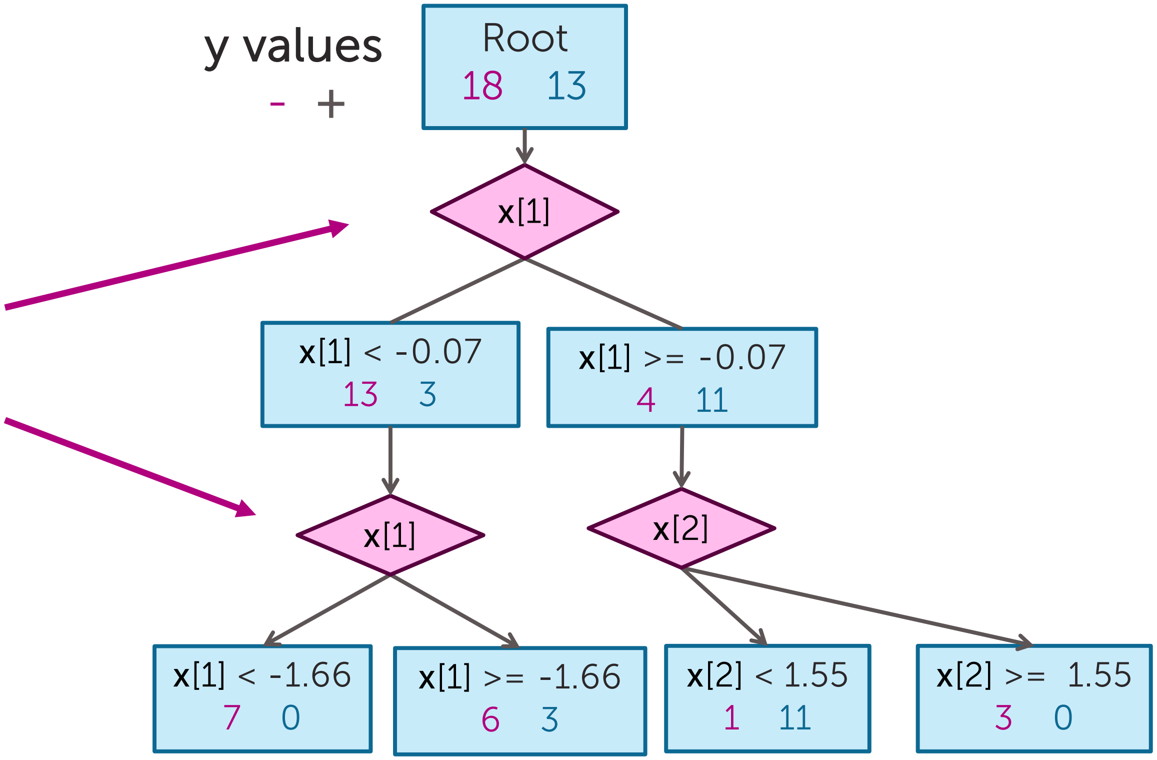
x[2] < 1.55

1 11

x[2] >= 1.55

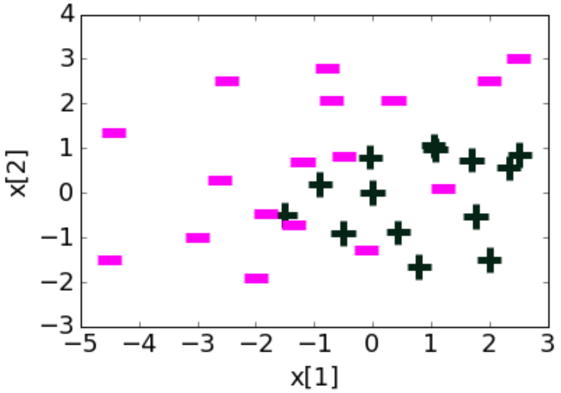
3 0

Threshold split caveat



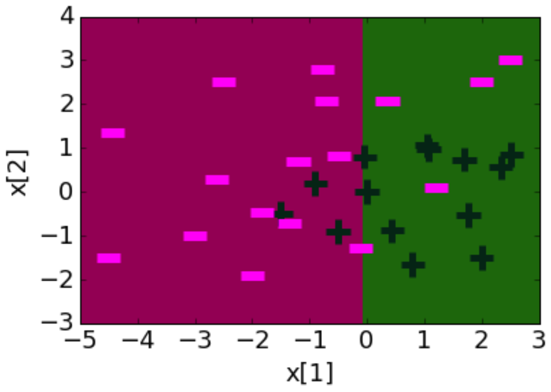
For threshold splits, same feature can be used multiple times

Decision boundaries

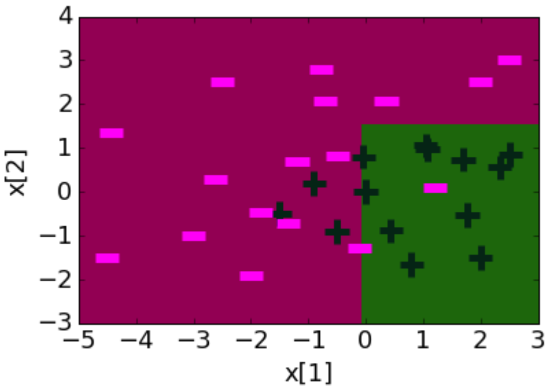


Overfitting ?

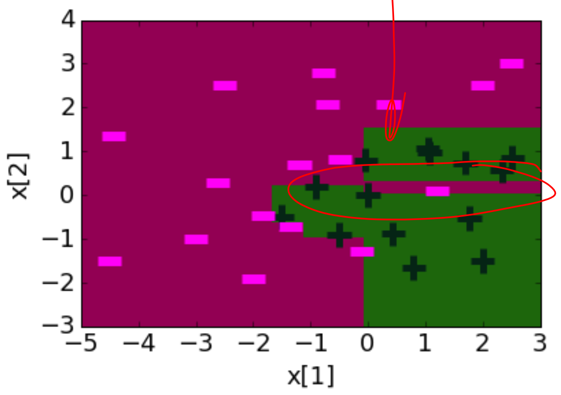
Depth 1



Depth 2



Depth 10



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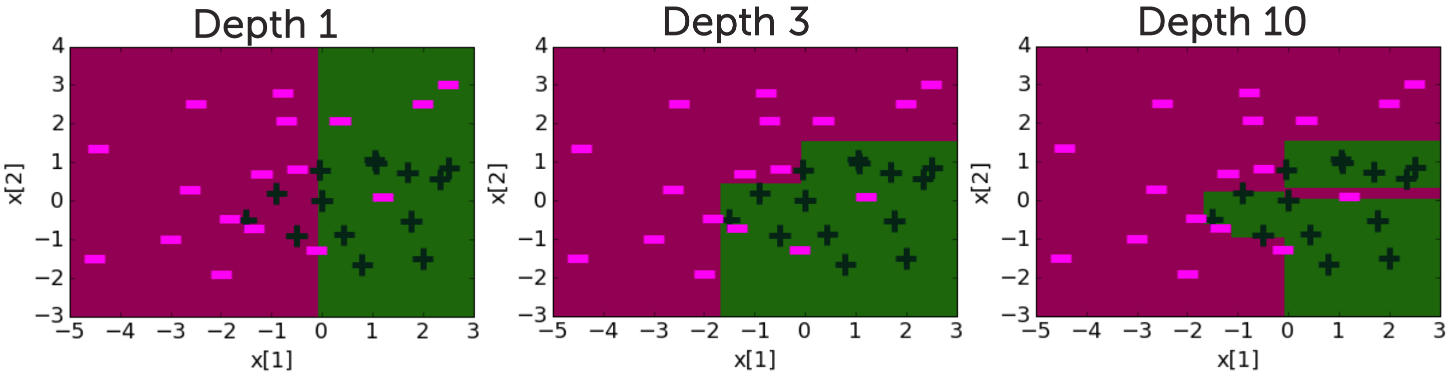
Comparing decision boundaries

Decision Tree

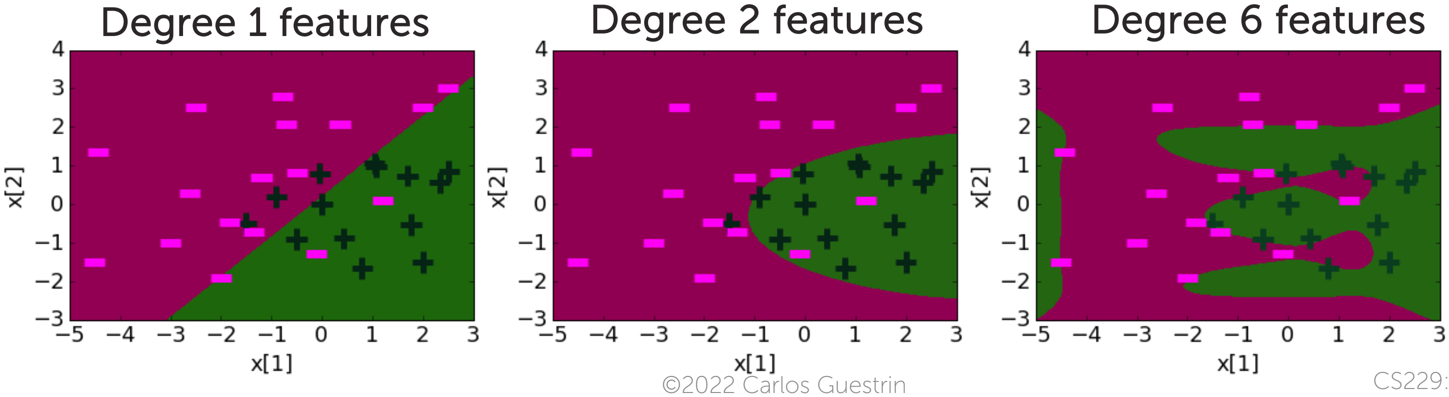


Depth 1

Depth 3

Depth 10

Logistic Regression



Degree 1 features

Degree 2 features

Degree 6 features

Summary of decision trees

What you can do now

- Define a decision tree classifier
- Interpret the output of a decision trees
- Learn a decision tree classifier using greedy algorithm
- Traverse a decision tree to make predictions
  - Majority class predictions
- Tackle continuous and discrete features