Non-stationary autoregressive filters for prediction of subsurface geological structure

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Geology's non-stationary issues

Accurate decision-making in the petroleum industry is highly contingent on building a reliable model that reasonably follows subsurface geological structure. A major challenge:

- Geology -> Highly heterogeneous

How can non-stationary prior information be incorporated for effectively predicting geology?

Learn and regularize

Subsurface models are built by solving inverse problems:

Data-fitting: $\mathbf{0} \approx \boldsymbol{r_d} = \boldsymbol{Fm} - \boldsymbol{d}$

Regularization: $\mathbf{0} \approx \boldsymbol{r_m} = \boldsymbol{Am}$

$\boldsymbol{F}$: Physical model, $\boldsymbol{m}$: Model parameters, $\boldsymbol{d}$: Data, $\boldsymbol{A}$: Regularization operator, $\boldsymbol{r_d}$ & $\boldsymbol{r_m}$: Residuals

Goal: Use machine learning to design a regularization operator which can learn non-stationary prior information

Non-stationary filters for non-stationary geology

Autoregressive filters can learn on prior information in the form of a training image (TI) for geology (Claerbout, 1999). To handle non-stationarity, a non-stationary approach is adopted -> multiple filters learn over the grid of a TI. To make the system of equations overdetermined, I assumed access to multiple TIs.

Mathematical formulation:

- 1D TIs -> $\boldsymbol{d_k} = \{d_{1k}, d_{2k}, d_{3k}\}, k = 1,..,n$
- 3×1 filter for every grid cell location. Filter for 3rd cell -> $\{1, a_{13}, a_{23}\}$
- System of equations for 3rd grid cell:

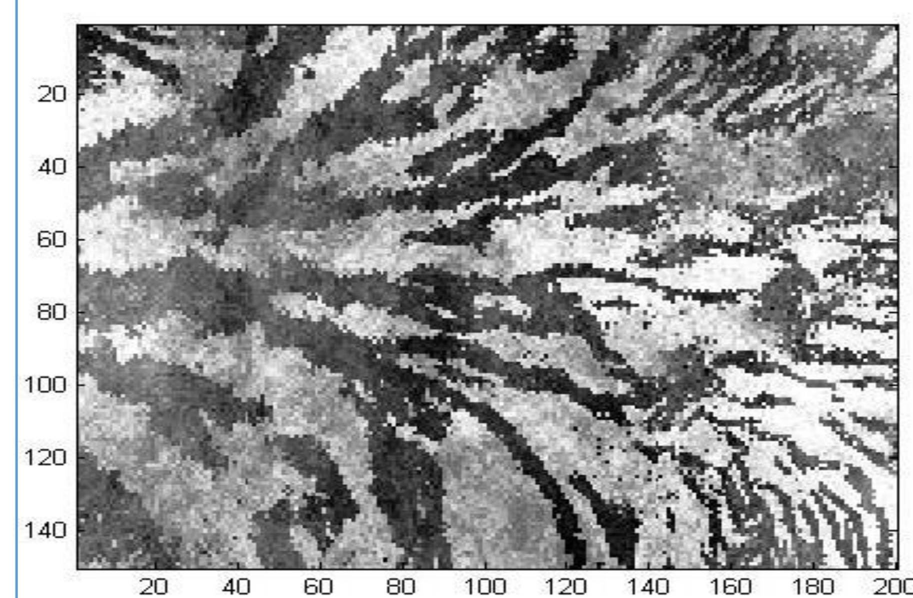
$$\mathbf{0} \approx \boldsymbol{r} = \begin{bmatrix} d_{31} & d_{21} & d_{11} \\ d_{32} & d_{22} & d_{21} \\ \vdots & \vdots & \vdots \\ d_{3n} & d_{2n} & d_{1n} \end{bmatrix} \begin{bmatrix} 1 \\ a_{13} \\ a_{23} \end{bmatrix}$$

- Regression equations formed for every cell and whole system optimized in a least squares sense

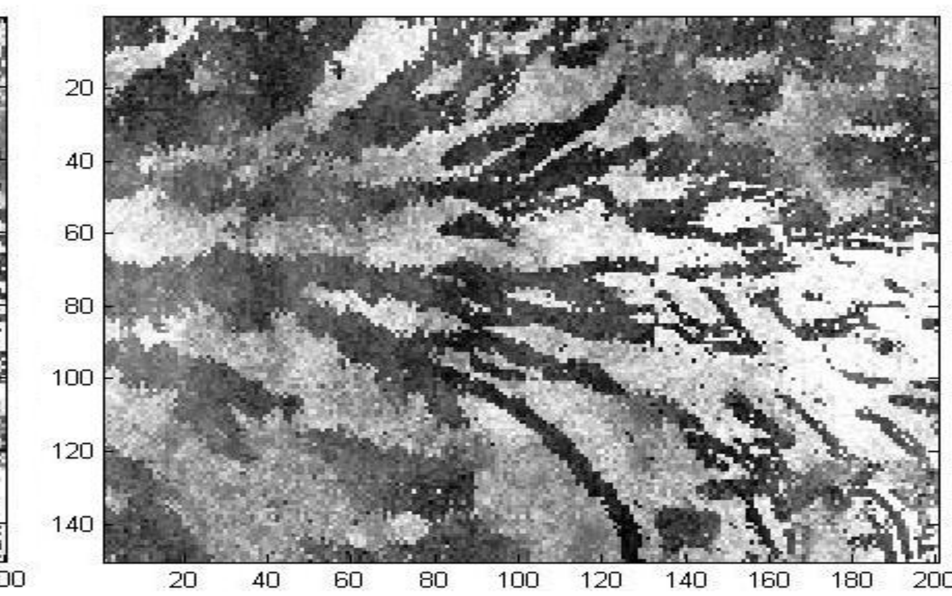
Data: Multiple TIs

Geostatistical simulations generate probable realizations of desired parameters. These serve as multiple TIs of the geological structure in my approach. I used Direct Sampling to generate TIs for two scenarios:

- A complex system of channel structures, commonly encountered in deltaic reservoirs.

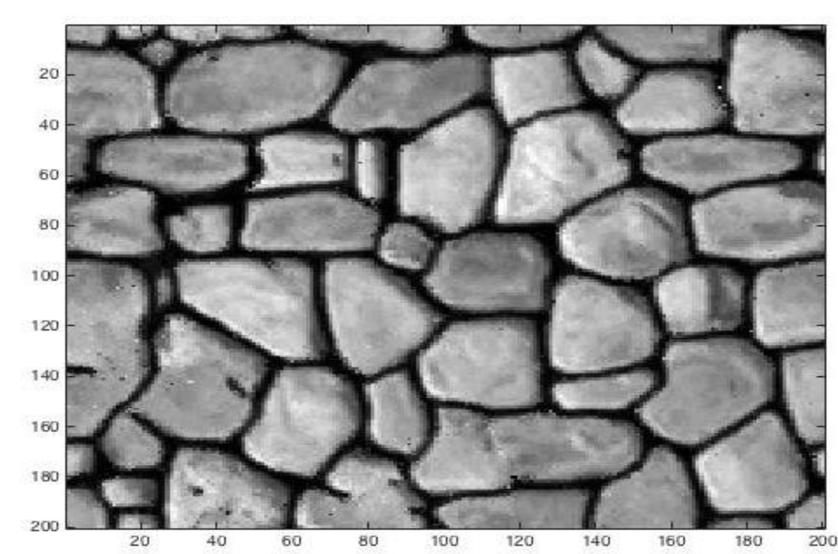


Realization 1

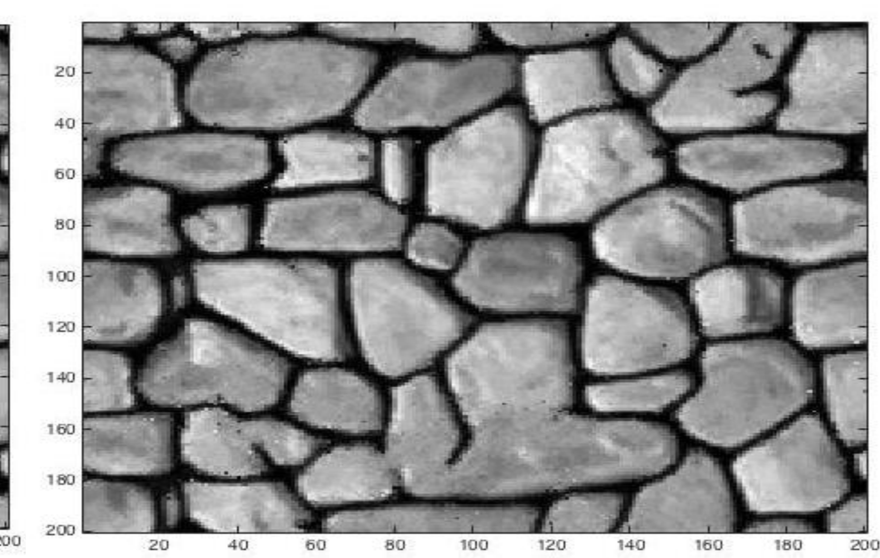


Realization 2

- A stonewall. Though not a geological scenario, the structure is highly non-stationary to be representative of geology.



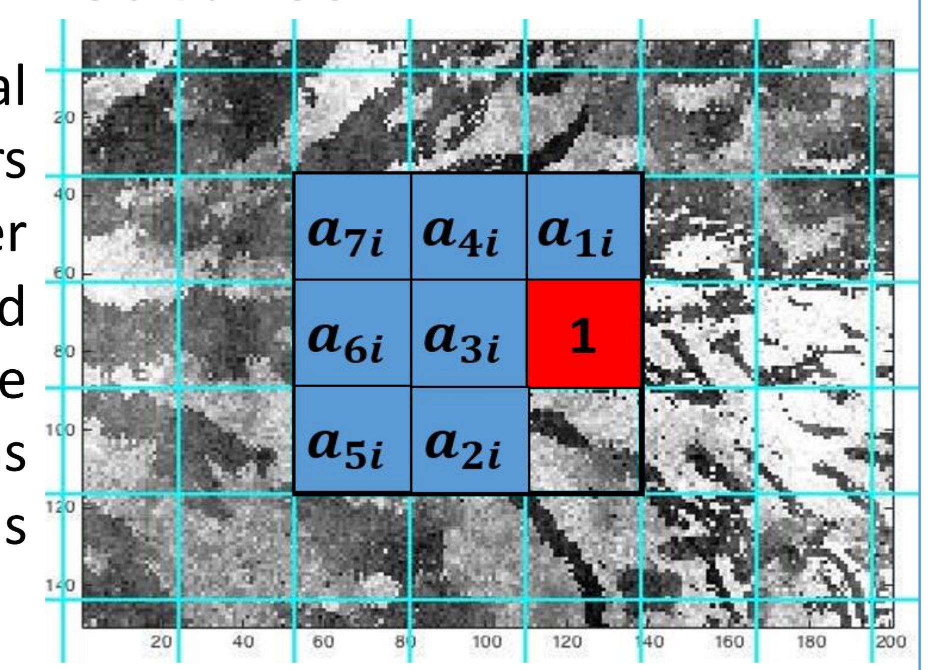
Realization 1



Realization 2

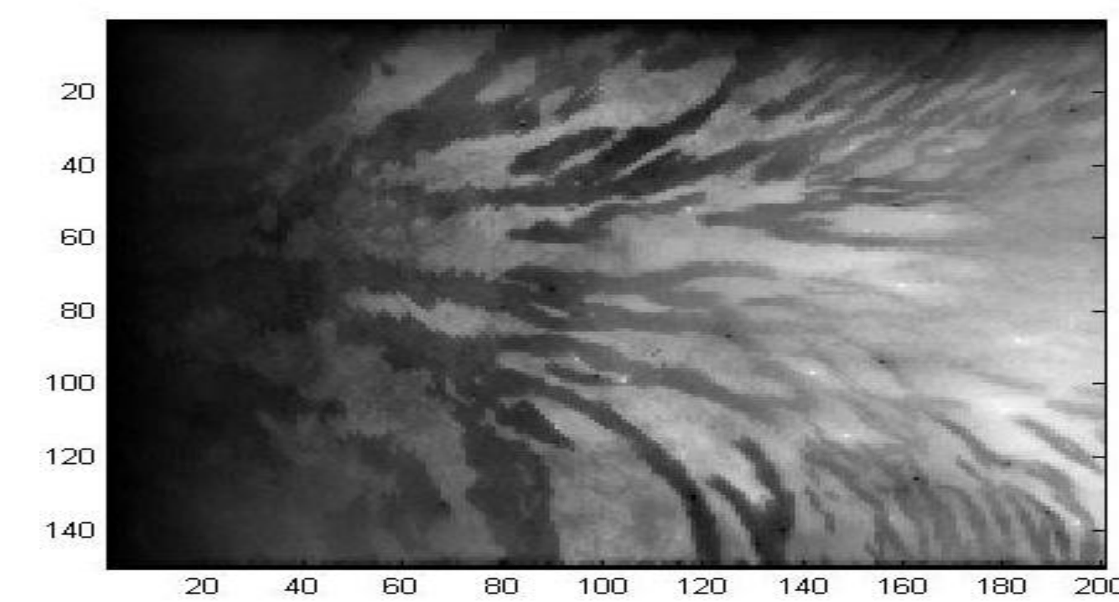
2D spatial features

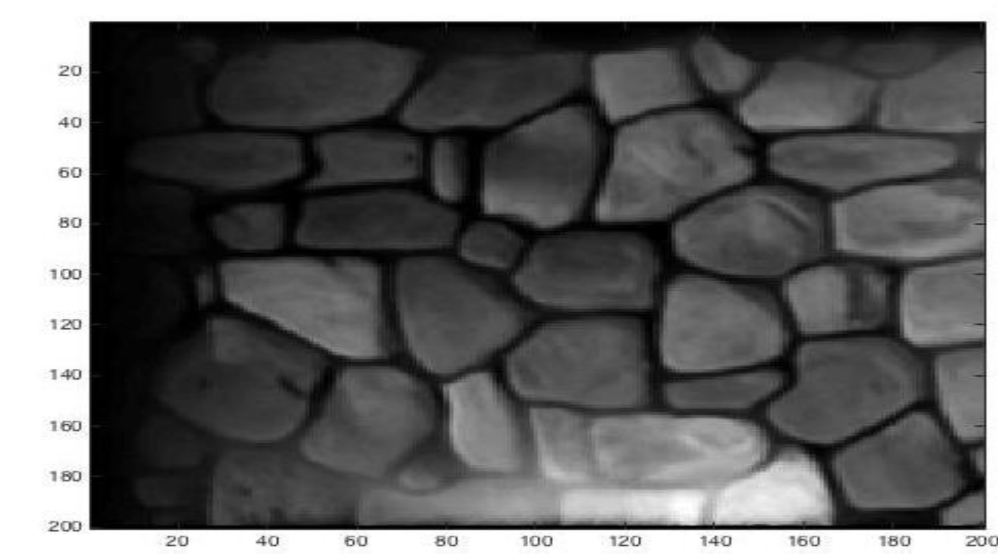
To effectively capture the spatial geological patterns, I use 2D filters (as shown alongside, a 3×3 filter for i^{th} cell). Thus, the features used in the regression equations are the spatially neighboring points. This helps capture the geological patterns effectively.



Results

2×2 Non-stationary filters were optimized in least squares sense using a conjugate gradient scheme. 100 TIs were used for each scenario. It is noted that autoregressive filters ideally have white output. Subsequently, for prediction, the grid was initialized with random numbers and the system was optimized in a reverse sense, i.e. model parameters were optimized. The results are depicted below:





Discussion

The non-stationary autoregressive filters were successful in capturing the highly heterogeneous geological patterns. When used as regularization operators, these filters are expected to guide inverse problems towards the desired geological solution, thus increasing confidence on high-stakes decisions involving drilling of wells.

Future work: Applying filters in 2D patches over the grid to minimize risk of overfitting.

Reference: Claerbout, J., 1999, Geophysical estimation by example: Environmental soundings image enhancement: SEP