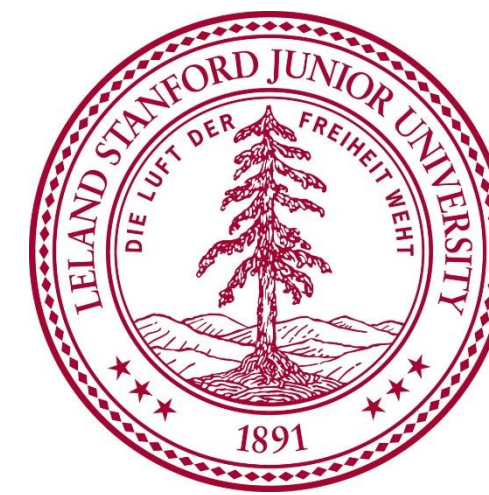




# GPS Pseudorange Error Characterization and Satellite Selection

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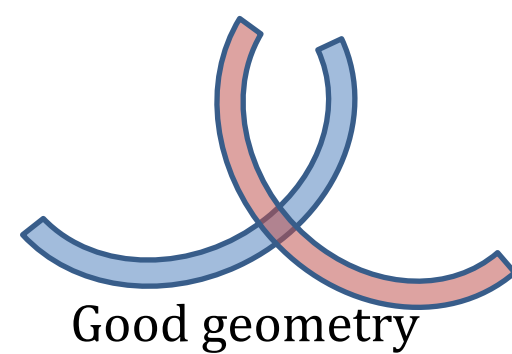
## Background

The GPS satellite constellation consists of only 32 satellites, but other satellite navigation constellations are being developed, and the total number of navigation satellites will grow to over 120 satellites. This number of new satellites will provide excellent geometric diversity, but an efficient receiver will only track and use a subset of the satellites in view for power and computation savings.

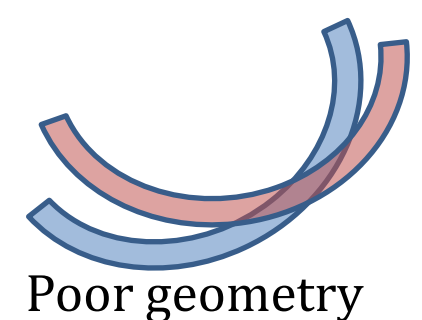
The goal of this work is to use machine learning techniques to efficiently predict the receiver performance using a subset of satellites in view in order to choose a well-performing subset.

## Satellite selection basics

### Geometric Diversity



Good geometry



Poor geometry

### Estimating error

#### User geometry matrix G:

Rows are made up of unit line of sight vectors to satellites

#### Satellite range err. cov. matrix R:

Ranging variances from each sat lie on diagonal of R

#### Covariance of position error:

$$P = (G^T R^{-1} G)^{-1} \quad [1]$$

#### RMS position error:

$$\epsilon = \text{tr}(P)^{\frac{1}{2}} \quad [1]$$

Costly to compute for each subset!

## Data source

Data source:

- International GNSS Service (IGS) GNSS receiver network[2]
- Observations from a single day from 20 Trimble NetR9 receivers
- Measurements:
  - Pseudorange- direct range measurement from sat.
  - Signal to noise ratio (SNR)- signal strength
  - User Range Accuracy (URA)- parameter broadcast by satellite indicating ranging accuracy

Truth ranging error:

- Precise Point Positioning service used to find truth rcvr. position and clock state -send in receiver observation files to service, receive precise receiver estimates
- Also receive ranging measurement residuals (errors)

## Data preprocessing/feature generation

In order to predict the aggregate position error  $\epsilon$ , we need the variance of the ranging error from each signal. We can find this empirically as a function of elevation angle, SNR, and URA by using a locally weighted standard deviation estimator given the measurement residuals from the PPP.

$$\sigma = \sqrt{\frac{1}{\sum w^{(i)}} \sum w^{(i)} (x^{(i)} - \mu)^2} \quad w^{(i)} = \exp\left(-\frac{(x^{(i)} - \mu)^2}{2\tau^2}\right)$$

At each epoch of the training data, take subset of satellites and compute position error variance given the geometry of the satellites and ranging error estimates. This serves as the truth data. We approach this as a classification problem, where acceptable subsets have an error below a certain threshold of  $\epsilon^2 < 15m$ . We use a subset size of 6 satellites.

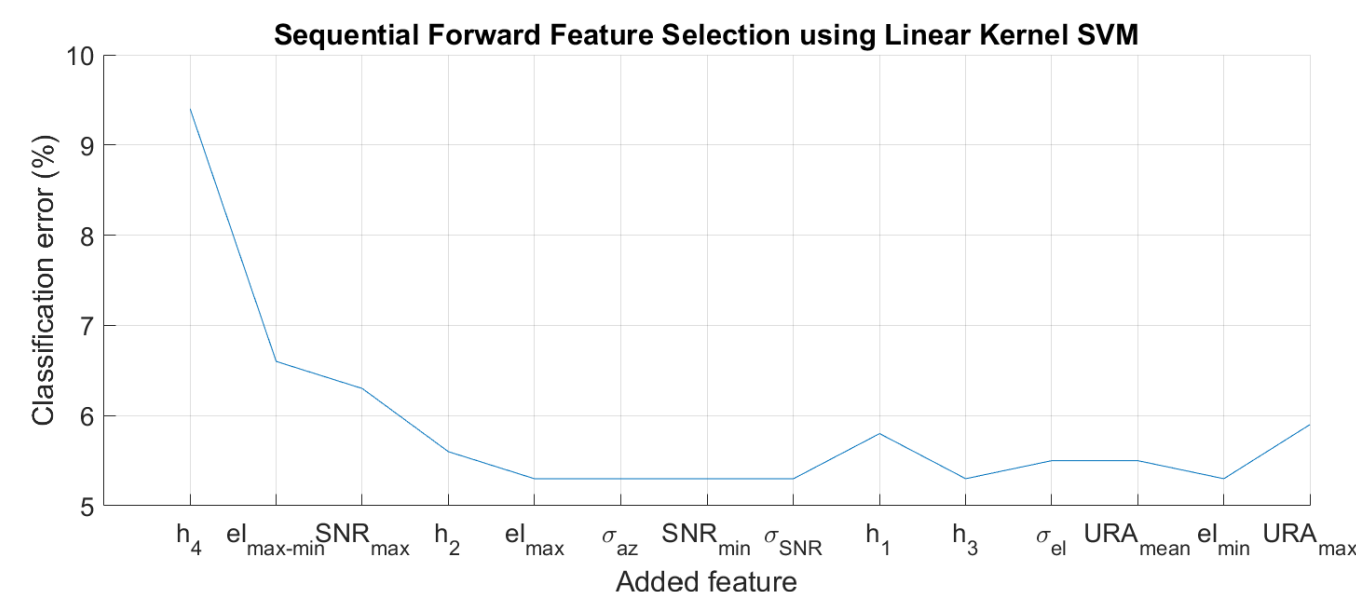
Simple features were selected based on available raw data- elevation angle to each satellite (el), azimuth (az), SNR, URA, and the geometry matrix (G).

## Overall input-output

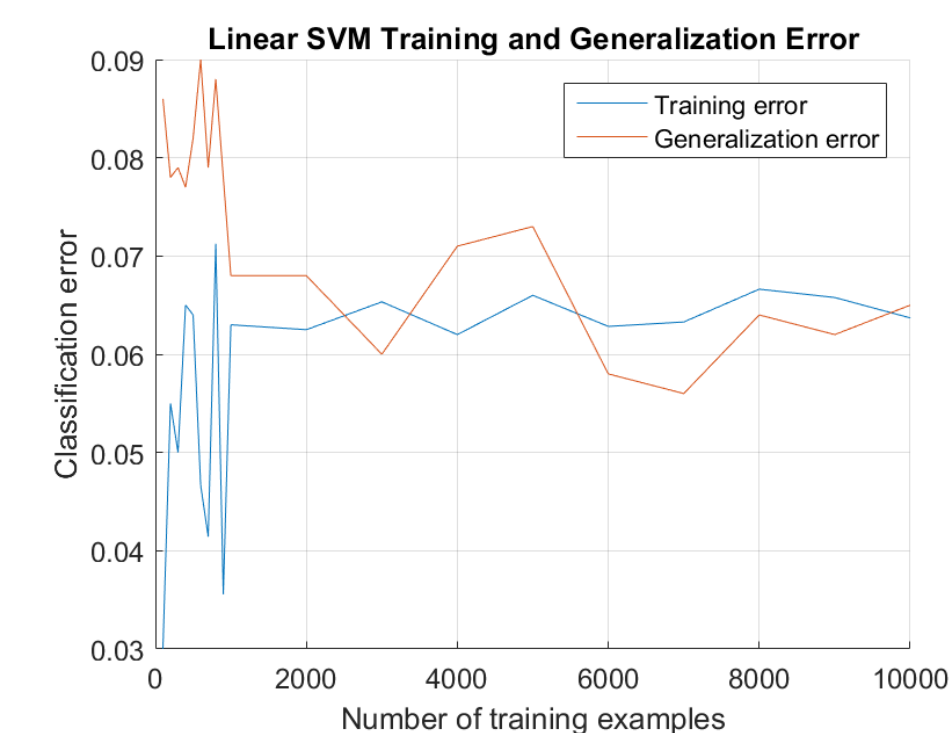
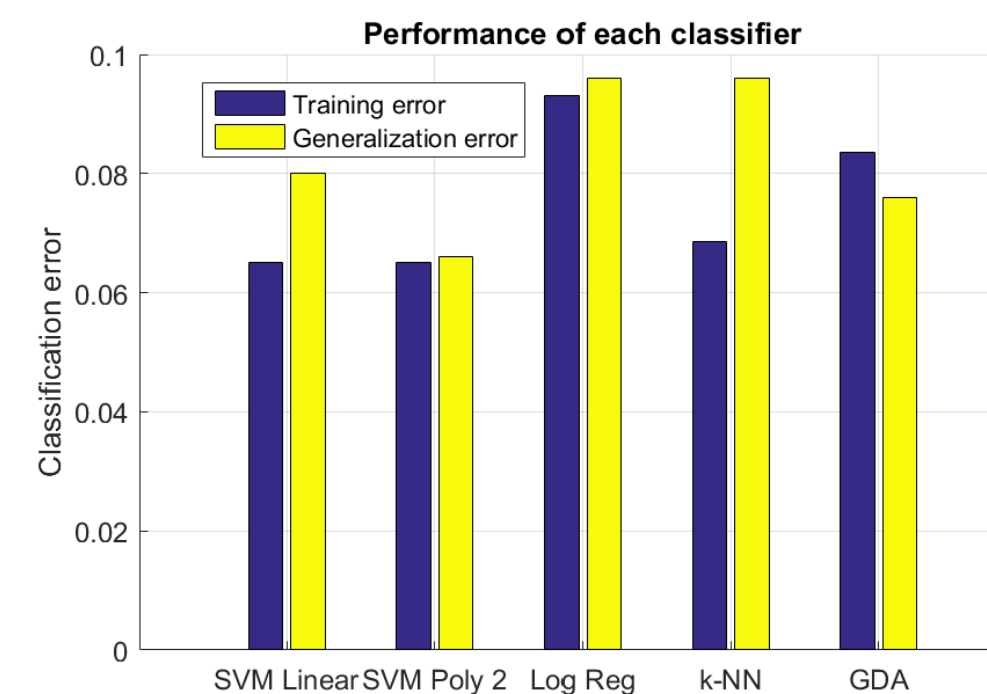
$$\begin{matrix} h_1 = \text{tr}(G^T G) \\ h_2 = \text{tr}((G^T G)^2) \\ h_3 = \text{tr}((G^T G)^3) \\ h_4 = |G^T G| \end{matrix} \quad [3] \quad \begin{matrix} \max(el) \\ \max(el) - \min(el) \\ \min(SNR) \\ \max(SNR) \\ \sigma_{el} \\ \sigma_{az} \\ \min(el) \end{matrix} \quad \begin{matrix} \max(SNR) \\ \min(SNR) \\ \sigma_{SNR} \\ \max(URA) \\ \min(URA) \end{matrix} \quad \rightarrow \quad \begin{matrix} \text{Satellite subset} \\ \text{classification- does} \\ \text{this subset meet our} \\ \text{accuracy} \\ \text{requirements?} \end{matrix}$$

## Feature selection

All features were normalized to lie between 0 and 1, and forward feature selection was used for each model, which improved performance in some cases and removed complexity overall. Geometry factors and SNR scored very highly on impact- URA had little impact on classification.

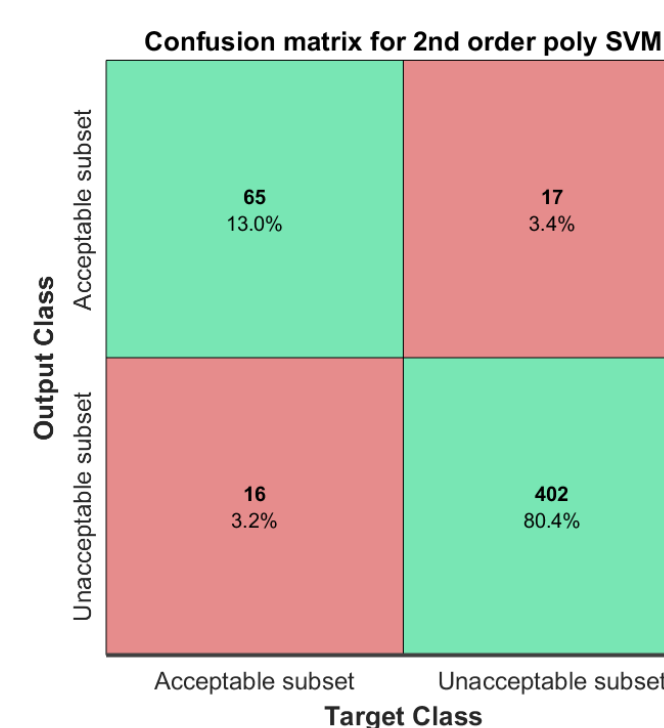


## Results and Analysis



Five classification methods were employed: support vector machine with linear kernel, support vector machine with second order polynomial kernel, logistic regression, k-nearest neighbors, and Gaussian discriminant analysis. The highest performer was the SVM using the second order polynomial kernel.

The confusion matrix shows that there is a somewhat problematic false positive rate, but further inspection shows that these false positives are all near the decision boundary and may be acceptable.



## Conclusions and Future Work

Machine learning techniques were used to accurately classify acceptable GNSS satellite subsets as below an error threshold. Prediction using these techniques are less computationally costly than a matrix inverse for each subset accuracy computation. Future work would be to include multiple GNSS constellations. Additionally, one might wish to take current features and propagate forward in order to do planning of satellite selection rather than the simple current time step implementation done here.

## References

- [1] P. Misra and P. Enge, *Global Positioning System: Signals, Measurements and Performance Second Edition*. Lincoln, MA: Ganga-Jamuna Press, 2006.
- [2] I. G. Service. *IGS Products*. Available: <http://www.igs.org/products>
- [3] D. Simon and H. El-Sherief, "Navigation satellite selection using neural networks," *Neurocomputing*, vol. 7, no. 3, pp. 247-258, 1995.