Learn to Integrate diagram and text in AI geometry reasoning

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Abstract

In this project, we simulated geometry concepts acquisition with unsupervised learning in deep neural network. Specifically, we built a model that learns to combine diagrams and text descriptions for geometry problems. In contrast to typical machine learning studies, we used a small number of unlabeled training samples and required the model to perform multiple tasks.

The results show that the network is able to learn several geometry concepts, including line lengths, angle sizes, line orientations, congruent triangles, etc. We also tested whether the model can generalize the learned geometry concepts to new shapes, and found that the generalization is very poor.

Using unsupervised deep learning, the current work provides a modeling framework to simulate how people learn to combine diagrams and text descriptions in geometry reasoning, which has significant implications for cognitive sciences.

Model Description

The unsupervised learning stage is implemented by a stack of restricted Boltzmann machines (RBMs).

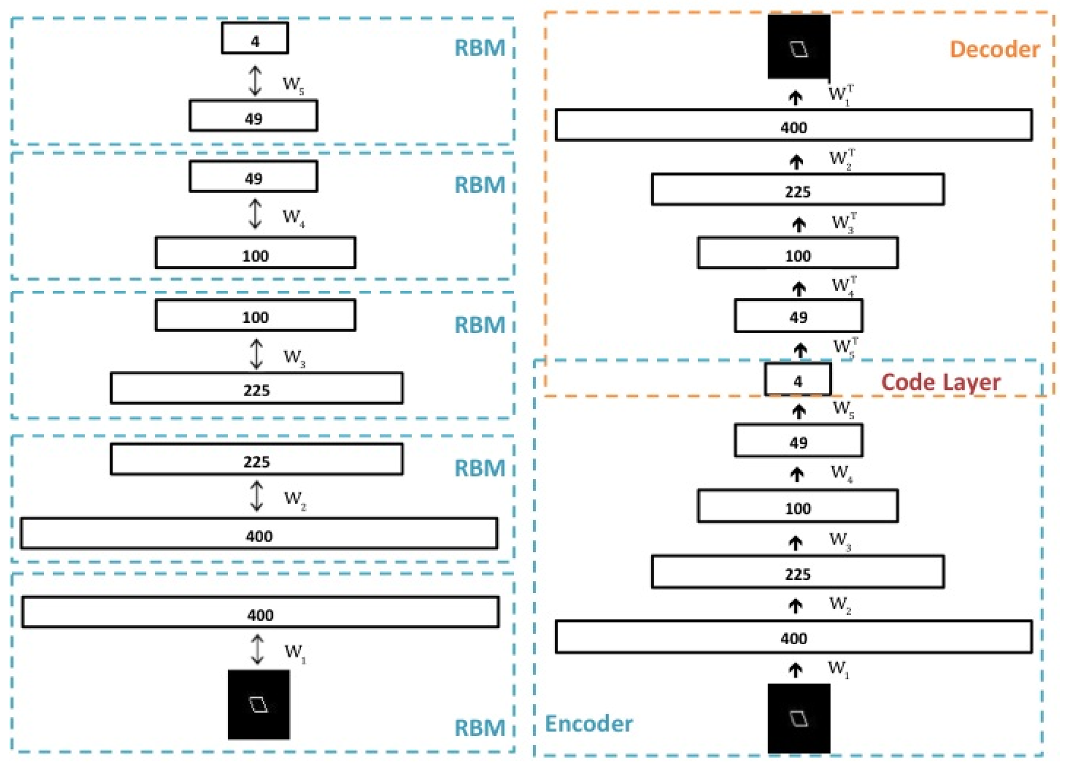


Figure 1 : Model Architecture

Given a training example, each binary node in the hidden layer is set to be 1 with probability

$$\sigma(b_j + \sum_i v_i w_{ij})$$

Once the binary states of the hidden nodes are all set, their activity is propagated back, i.e., each node in the input layer is set to 1 with probability

$$\sigma(b_i + \sum_j h_j w_{ij})$$

After this reconstruction phase, the hidden layer activity is calculated once more using the reconstructed input, and change of weights follows

$$\triangle w_{ij} = \alpha(< v_i h_j >_{\text{data}} - < v_i h_j >_{\text{recon}}).$$

Results

I. The learning of geometry concepts

(A) Magnitude comparison.

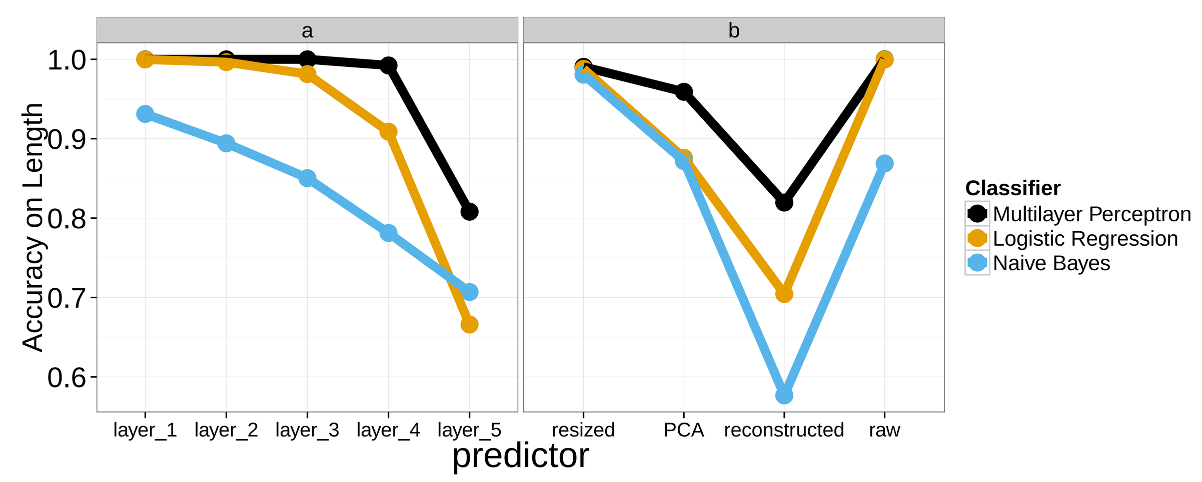


Figure 2 : Accuracy on comparing the lengths of two lines

(B) Spatial relationship regarding rotation.

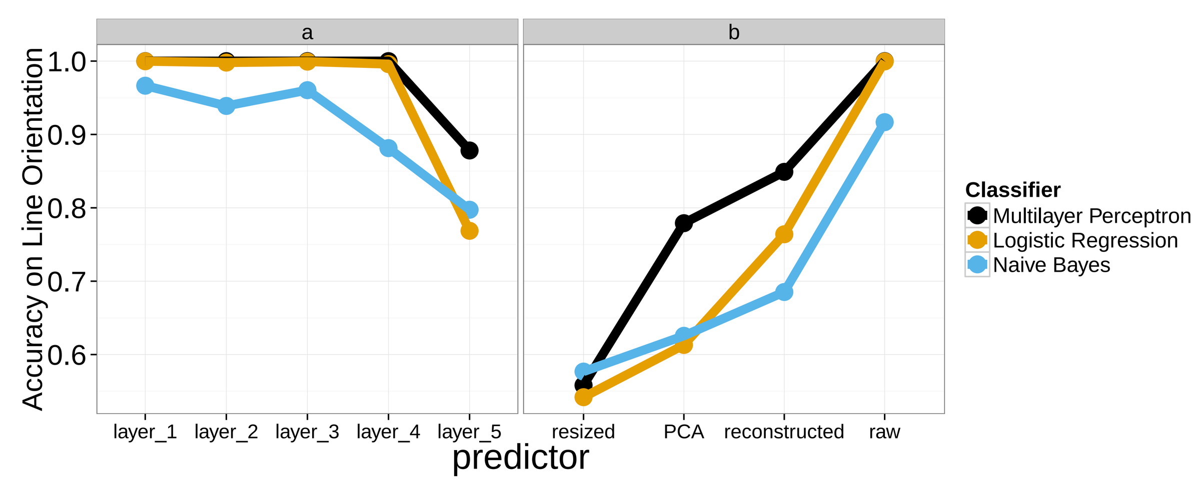


Figure 3 : Accuracy on comparing the orientations of two lines

(C) Congruent triangles.

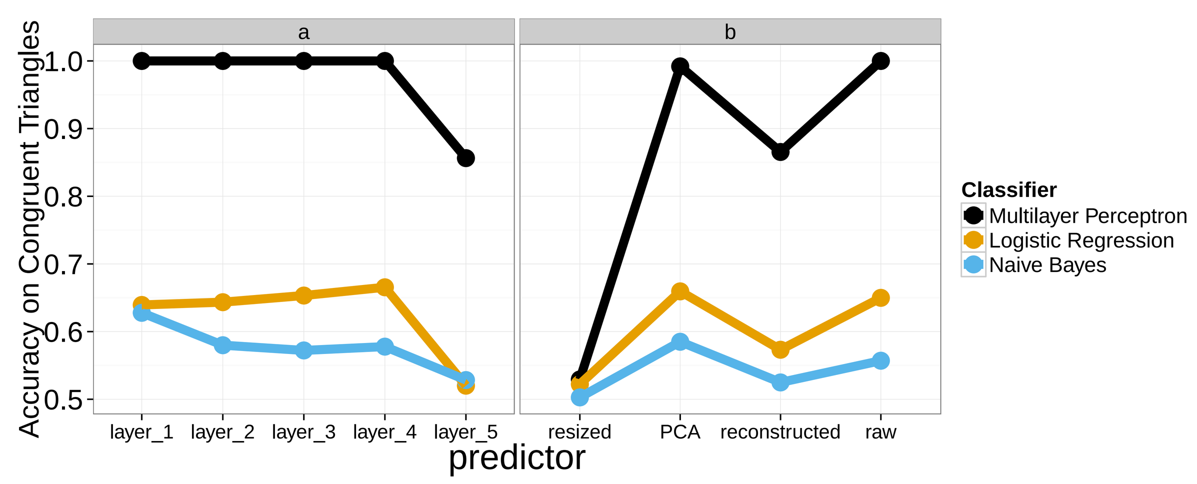


Figure 4 : Accuracy on judging whether two triangles are congruent or not

II. The generalization of learned concepts

(A) The generalization of magnitude comparison. The classifiers are trained to perform the line length comparison task.

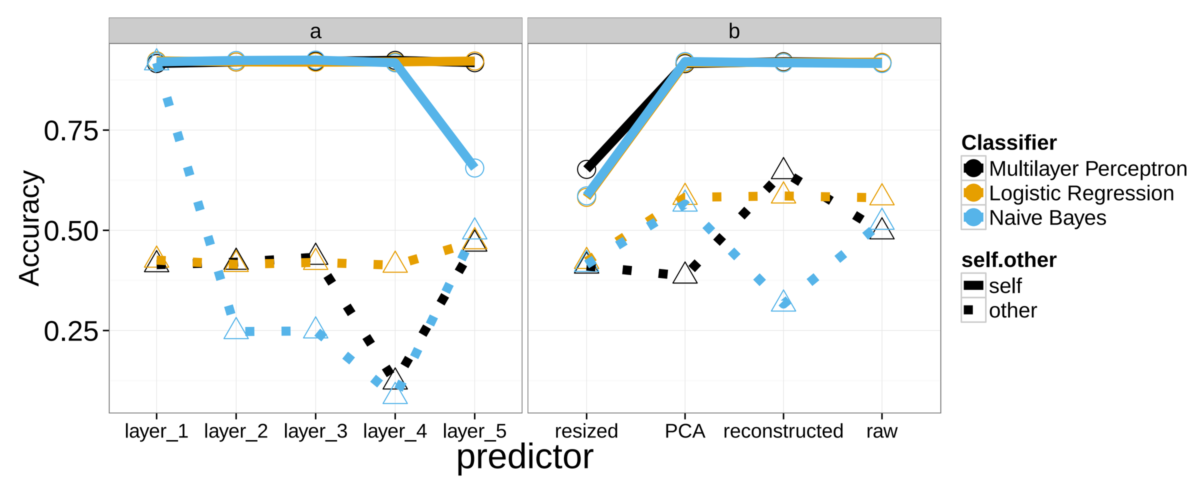


Figure 5 : Accuracy on comparing the sizes of two circles.

(B) The generalization of congruency. The classifiers are trained to judge whether two triangles are congruent or not.

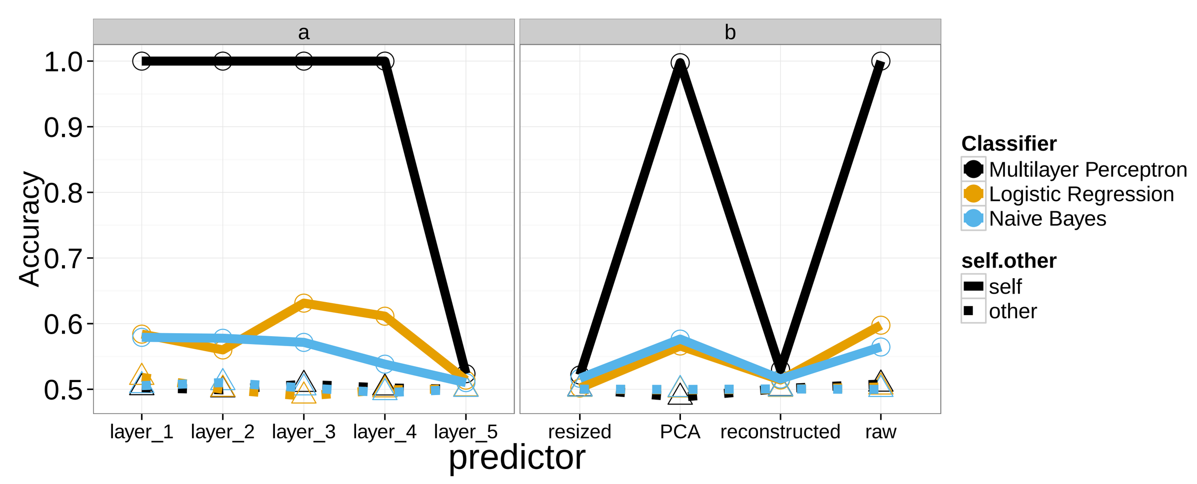


Figure 6 : Accuracy on judging whether two quadrilaterals are "congruent" or not

(C) The generalization of rotation. The classifiers are trained to compare the orientations of two lines.

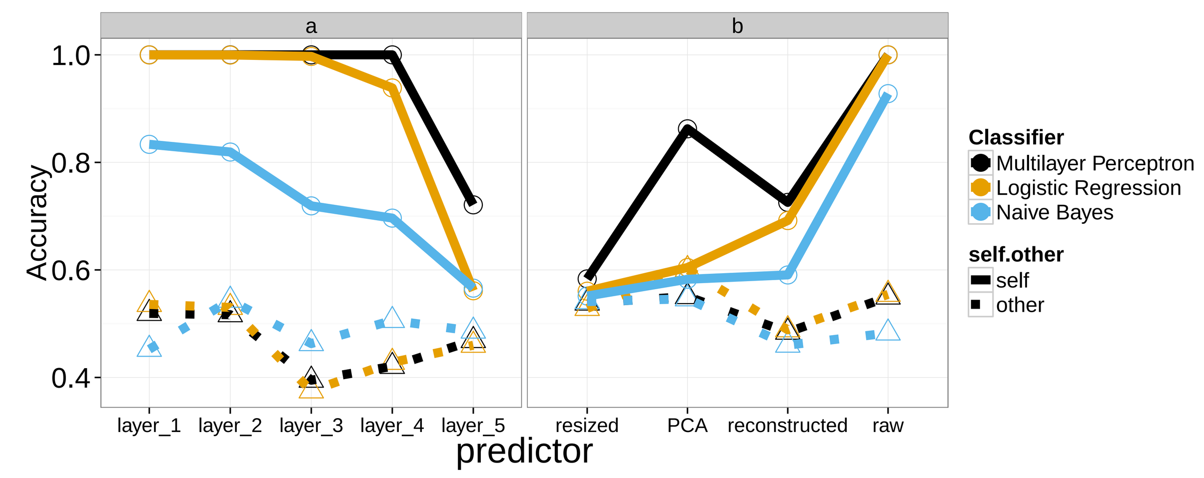
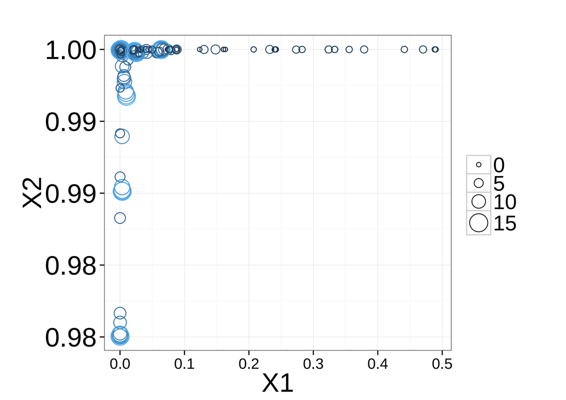
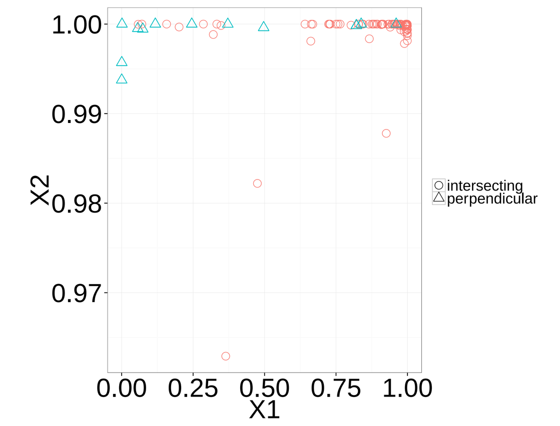


Figure 7 : Accuracy on judging the orientation of two parallel lines

III. The visualization of the learned concepts



(a) Line lengths



(b) Perpendicular vs. intersecting lines

Conclusions

- Deep network can extract essential information in geometry figures with unsupervised learning.
- Hidden units in the trained neural network does not have the same properties as observed in neurons of human visual cortex.
- Without labeled training samples, learned geometry concepts does not generalize to new shapes.

References

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Zorzi, M., Testolin, A., and Stoianov, I. P. (2013). Modeling language and cognition with deep unsupervised learning: a tutorial overview. *Frontiers in psychology*, 4.