Experimenting with Algorithmic Composition Techniques

Rondo Alla Turca (Excerpt)
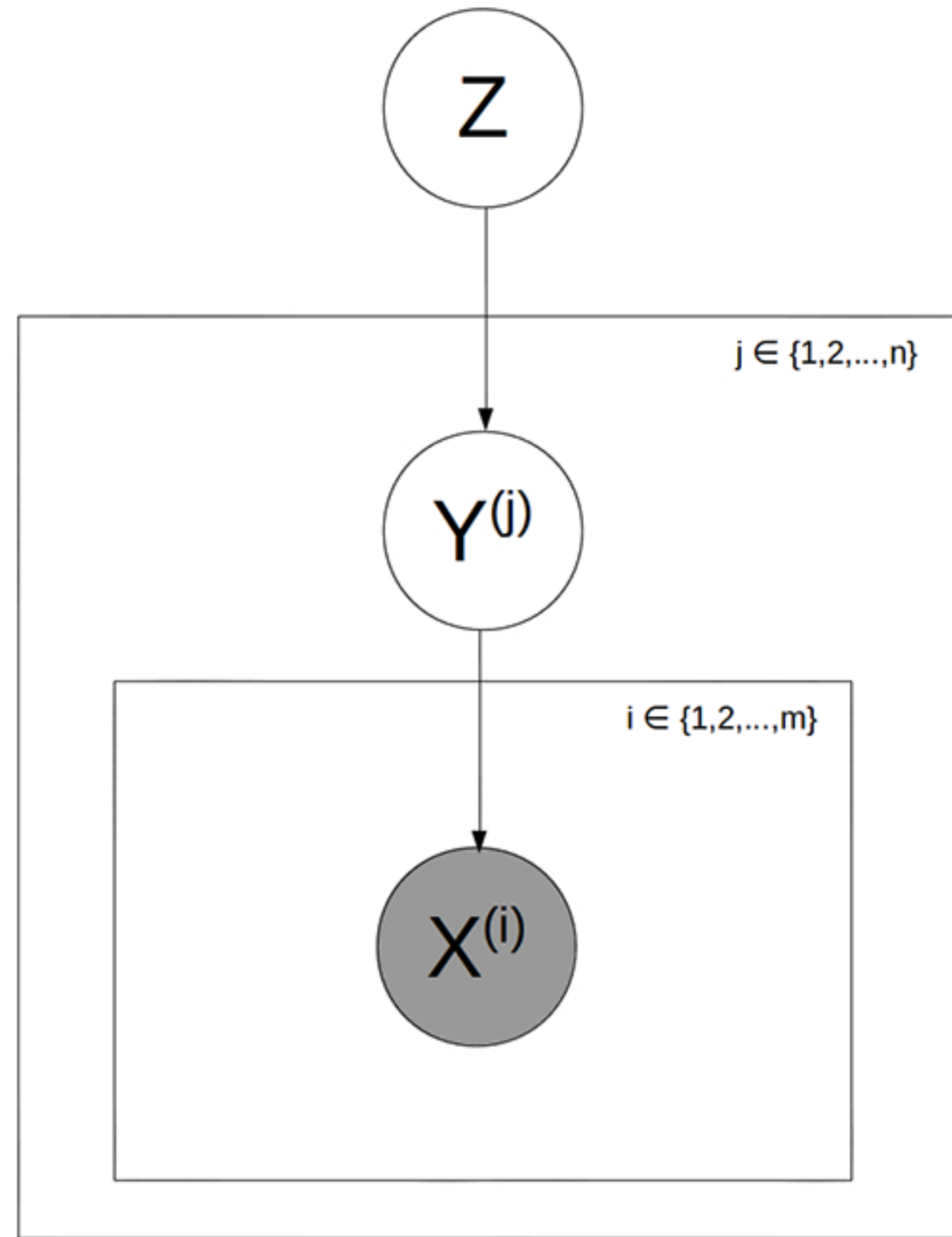
from Piano Sonata No. 11, K. 331

W. A. Mozart

Allegretto

A Major

F# Minor

```
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    notated_32nd_notes_per_beat=8 time=0>
<meta message set_tempo tempo=500000 time=0>
<meta message key_signature key='C' time=0>
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note_off channel=0 note=71 velocity=0 time=92
note_on channel=0 note=69 velocity=59 time=4
note_off channel=0 note=69 velocity=0 time=92
note_on channel=0 note=68 velocity=59 time=4
note_off channel=0 note=68 velocity=0 time=92
note_on channel=0 note=69 velocity=59 time=4
note_off channel=0 note=69 velocity=0 time=92
note_on channel=0 note=72 velocity=59 time=4
note_off channel=0 note=72 velocity=0 time=188
```

Z

$j \in \{1,2,\dots,n\}$

$Y^{(j)}$

$i \in \{1,2,\dots,m\}$

$X^{(i)}$

$Z \sim Categorical(\theta) = \prod_{\alpha=1}^{24} \theta_\alpha^{[z=\alpha]}$

$Y^{(j)}\big| z \sim Categorical(\Upsilon_z) = \prod_{\beta=1}^{24} \Upsilon_{z,\beta}^{[y^{(j)}=\beta]}$

$X^{(i)}\big| y^{(j)} \sim Categorical(\Psi_{y^{(j)}}) = \prod_{\lambda=1}^{88} \Psi_{y^{(j)},\lambda}^{[x^{(i)}=\lambda]}$

E Step

$$q^{(t)}\big(y^{(j)}, z\big|x^{(i)}\big) := p\big(y^{(j)}, z\big|x^{(i)}; \theta^{(t-1)}, \Upsilon^{(t-1)}, \Psi^{(t-1)}\big)$$

M Step

$$\sum_{\ell=1}^{m}\sum_{y^{(\ell)}}\sum_{z^{(\ell)}} q\big(y^{(\ell)}, z^{(\ell)}\big|x^{(\ell)}\big) \log p\big(x^{(\ell)}, y^{(\ell)}, z^{(\ell)}; \theta, \Upsilon, \Psi\big)$$

$$= \sum_{\ell=1}^{m}\sum_{y^{(\ell)}}\sum_{z^{(\ell)}} q\big(y^{(\ell)}, z^{(\ell)}\big|x^{(\ell)}\big) \log p\big(x^{(\ell)}\big|y^{(\ell)}; \Psi\big) p\big(y^{(\ell)}\big|z^{(\ell)}; \Upsilon\big) p\big(z^{(\ell)}; \theta\big)$$

$$= \sum_{\ell=1}^{m}\sum_{y^{(\ell)}}\sum_{z^{(\ell)}} q\big(y^{(\ell)}, z^{(\ell)}\big|x^{(\ell)}\big) \log \left(\prod_{\alpha=1}^{88} \Psi_{y^{(\ell)},\alpha}^{[x^{(\ell)}=\alpha]} \prod_{\beta=1}^{24} \Upsilon_{z,\beta}^{[y^{(\ell)}=\beta]} \prod_{\beta=1}^{24} \theta_\beta^{[z^{(\ell)}=\beta]}\right)$$

$$= \sum_{\ell=1}^{m}\sum_{y^{(\ell)}}\sum_{z^{(\ell)}} q\big(y^{(\ell)}, z^{(\ell)}\big|x^{(\ell)}\big) \left(\sum_{\alpha=1}^{88} \log \Psi_{y^{(\ell)},\alpha}^{[x^{(\ell)}=\alpha]} + \sum_{\beta=1}^{24} \log \Upsilon_{z,\beta}^{[y^{(\ell)}=\beta]} + \sum_{\beta=1}^{24} \log \theta_\beta^{[z^{(\ell)}=\beta]}\right)$$

$$\theta^{(t)} := \arg\max_\theta \sum_{i=1}^{m}\sum_{y^{(j)}}\sum_{z} q^{(t)}\big(y^{(j)}, z\big|x^{(i)}\big) \log p\big(x^{(i)}, y^{(j)}, z; \theta, \Upsilon, \Psi\big)$$

$$\Upsilon^{(t)} := \arg\max_\Upsilon \sum_{i=1}^{m}\sum_{y^{(j)}}\sum_{z} q^{(t)}\big(y^{(j)}, z\big|x^{(i)}\big) \log p\big(x^{(i)}, y^{(j)}, z; \theta, \Upsilon, \Psi\big)$$

$$\Psi^{(t)} := \arg\max_\Psi \sum_{i=1}^{m}\sum_{y^{(j)}}\sum_{z} q^{(t)}\big(y^{(j)}, z\big|x^{(i)}\big) \log p\big(x^{(i)}, y^{(j)}, z; \theta, \Upsilon, \Psi\big)$$

Incomplete Log Likelihood vs Number of Iterations

log likelihood

Iterations EM