

# Distributed power control in asymmetric interference-limited networks

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## I. INTRODUCTION

Power control in wireless communication networks reduces interference by regulating access to the shared wireless medium and enhances the overall network capacity as well as battery life of nodes. Beginning with the Foschini-Miljanic (FM) algorithm [1], distributed power control for wireless networks has been an active area of research. The problem of distributed power control for packetized traffic was first addressed by Kandukuri and Bambos [2] by proposing “Power-controlled multiple access” (PCMA) family of algorithms, and subsequently investigated in [3], [5], [6]. PCMA algorithm uses dynamic programming approach to balance the power versus delay cost of the transmitter. It introduces three separate regimes: aggressive, soft backoff, hard backoff. In the low-interference regime, transmitter power tries to aggressively match and overcome the interference – up to a certain point, while in the high interference regime, the power is set to zero (hard backoff) and the transmitter ceases to transmit waiting for the low interference periods. Thus, PCMA algorithms usually induce a soft time-division multiple access (iTDMA) allowing links facing high interference to temporarily reduce their power and incur a delay cost instead. On the other hand, in Foschini-Miljanic (FM) algorithm, each link attempts to maintain a target SIR by overcoming the interference presented by the other links.

In this project, distributed power control algorithms are investigated in a two-link interference-limited network with asymmetric interference. Such asymmetric interference can occur in any cellular as well as adhoc wireless networks, when the first link receives strong interference from the second link, and the second link receives weak interference from the first link. For example, a downlink user connected to a distant macro-cell can be jammed due to the presence of a closer downlink user connected to a femtocell using the same frequency/time slot (see Fig. 1(a)). In adhoc wireless networks, the random placement of links may also result in asymmetric interference. Since distributed power control algorithms control transmitter power based on the interference seen by the transmitter, they can perform poorly in asymmetric interference-limited networks. If FM algorithm is used in asymmetric interference-limited networks, the weak link uses high power proportional to the interference from the strong link to maintain its target SIR, and its queue starts exploding at very low loads. PCMA algorithm performs better than FM algorithm in asymmetric interference-limited network, but it is unable to induce a iTDMA effect because the strong link can get stuck in aggressive or soft backoff regime when the average load is high and interference seen is low.

Distributed power control algorithms have not been investigated in asymmetric interference-limited networks to the best of my knowledge. In this project, distributed power control algorithms [1], [3] are simulated, and their performance is analyzed for two-link asymmetric interference-limited network. Based on the results, some variations are proposed to improve the performance of PCMA algorithms, and these variations are still under investigation.

## II. WIRELESS SYSTEM MODEL

Consider a channel with two interfering wireless links, and let  $g_{jk}$  be the power gain from the transmitter of link  $k$  to the receiver of link  $j$ . Let the first link in Fig. 1(b) receive strong interference from the second link (i.e.  $g_{11} < g_{12}$ ) and be referred to as *weak* link. Let the second link receive weak interference from the first link (i.e.  $g_{22} > g_{21}$ ) and be referred to as *strong* link. The link quality of service is denoted by the signal-to-interference-noise ratio (SINR), which is defined for link  $j$  as

$$\gamma_j = \frac{p_j g_{jj}}{\sum_{k \neq j} p_k g_{jk} + \eta_j} = \frac{p_j g_{jj}}{i_j} \quad (1)$$

where  $p_j$  is the transmit power of link  $j$ ,  $\eta_j > 0$  is the thermal noise power of the receiver of link  $j$ , and  $i_j$  is simply the total interference at the receiver of link  $j$ . The time is assumed to be slotted (indexed by  $n = 0, 1, \dots$ ), and each link is transmitting packetized traffic. The transmitter of each link  $j$  is equipped with a FIFO buffer, and this buffer contains  $b_j(n)$  packets at the beginning of time slot  $n$ . In every time slot, one packet is removed from the transmitter buffer (i.e., successfully transmitted to the receiver) of link  $j$  with success probability  $s(p_j(n), i_j(n))$ , where  $s(\cdot)$  is an increasing function of  $\gamma_j(n)$  (the link SINR at time  $n$ ). In the event of an unsuccessful transmission of a packet, the transmitter will continue to attempt to transmit the packet in subsequent time slots, until it is successfully received at the receiver. If the buffer is empty at the beginning of time slot  $n$ , then no transmission is attempted. Finally, in each time slot, a packet arrives in the transmitter buffer

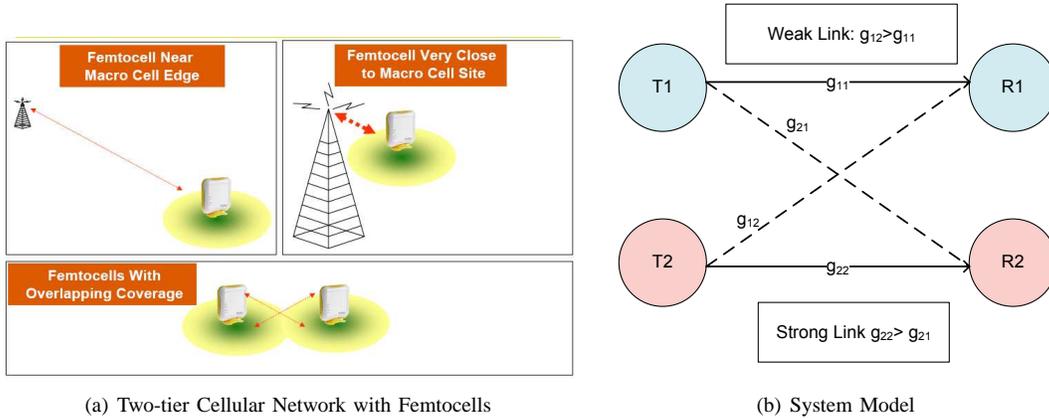


Fig. 1. Examples of Asymmetric Interference in Wireless Networks

of each link  $j$  with probability  $\lambda_j$  (after the transmission for that time slot has already been attempted) - this is the average load. For simplicity, the transmitter buffers are assumed to have infinite length and the transmit powers are continuous and bounded above by  $P_{max}$ .

### III. REVIEW OF DISTRIBUTED POWER CONTROL ALGORITHMS

*Foschini-Miljanic Algorithm:* In Foschini-Miljanic algorithm [1], a certain target SIR  $\gamma_t$  is chosen such that there exists a feasible set of power levels for all the links in the wireless network [4]. Assuming that each link can observe the interference at its receiver in each time slot, then each link iteratively sets its transmit power equal to the amount required to achieve the target SIR. So the power-update for link  $j$  in time-slot  $(n + 1)$  is given by –

$$p_j(n + 1) = \begin{cases} \frac{\gamma_t}{g_{jj}} i_j(n) & b_j(n) \geq 0 \\ 0 & b_j(n) = 0. \end{cases} \quad (2)$$

Foschini and Miljanic [1] showed that as long as there exists a feasible power vector for all links in the network (the conditions described in [4]), then this simple iterative algorithm will always result in exponentially fast convergence.

*Power-Controlled Multiple Access (PCMA) Algorithm:* Power-Controlled Multiple Access (PCMA) algorithms [3] are backlog-aware distributed power control algorithms proposed for delay-tolerant traffic to balance the trade-off between power and delay. The transmitter realizes that during high interference in the channel, it will have to use high power to overcome the interference and transmit a packet successfully to the receiver and so, it considers backing off and buffering the incoming traffic until the interference subsides, resulting in increased delay but reduced power. However, when it has backed off, its buffer begins to fill up, putting pressure on the transmitter to be aggressive in order to rapidly reduce its backlog, resulting in increased power but reduced delay.

PCMA addresses this power-delay trade-off using a dynamic programming approach to find an algorithm that minimizes the average overall incurred cost when a single communication link operates in a channel with extraneously driven random interference (unresponsive in nature). The overall cost of transmission in each slot is the power cost  $p_n$ , the power (actually energy) spent in the slot, and the backlog cost  $B(b_n)$ .  $B(\cdot)$  is a positive function that increases with the backlog  $b_n$ . Each link knows its current interference level  $i_n$ , and its current backlog  $b_n$  in each time slot. The system is allowed to evolve until all  $K$  packets have successfully been transmitted and the buffer becomes empty (assuming no packets arrive). Let  $V(b, i)$  be the cost-to-go, that is, the minimal expected cost that will be incurred under optimal power control until the buffer empties, if the system starts with backlog  $b$  and interference  $i$ . The standard dynamic programming (DP) recursion satisfied by  $V(b, i)$  is –

$$V(b, i) = \inf_{p \geq 0} \{ p + B(b) + s(p, i) \sum_{j \in \mathcal{I}} q_{ij} V(b - 1, j) + (1 - s(p, i)) \sum_{j \in \mathcal{I}} q_{ij} V(b, j) \}, \quad (3)$$

where  $q_{ij}$  represents the transition probabilities  $P[i_{(n+1)} = j | i_n = i] = q_{ij}$  for a time-homogeneous irreducible Markov chain over a finite set  $\mathcal{I}$  of interference states,  $s(p, i)$  is the success probability of a packet (increasing function of  $p$  and decreasing function of  $i$ ), and  $V(0, i) = 0$  for all  $i \in \mathcal{I}$ . The Markov chain over the interference states has a stationary distribution  $\pi_i$  at state  $i \in \mathcal{I}$  when the interference levels in various slots are i.i.d. Then the dynamic programming recursion can be written as –

$$V(b, i) = \inf_{p \geq 0} \{ p - s(p, i) X(b) + Y(b) \}, \quad (4)$$

where

$$X(b) = \sum_{j \in \mathcal{I}} \pi_j [V(b, j) - V(b-1, j)], \quad (5)$$

and

$$Y(b) = \sum_{j \in \mathcal{I}} \pi_j [B(b) + V(b, j)]. \quad (6)$$

Based on the structural properties of this dynamic programming recursion, it is observed that under low interference, it is effective to transmit, and thereby reduce the backlog/delay cost by incurring a moderate power cost. However, as the interference goes up, the probability of a successful transmission decreases, and it becomes preferable to incur some delay cost to avoid an excessive power cost. As the backlog increases, the link must transmit more aggressively (higher power). This is concretely defined for two different functional forms of  $s(p, i)$ , though the tradeoff holds true for any functional form of  $s(p, i)$  (increasing in  $p$  and decreasing in  $i$ ).

- PCMA-1: For transmission success probability  $s^1(p, i) = \frac{p}{\alpha p + \beta i}$  and i.i.d interference in each time slot, the optimal power update policy is given by

$$p^{*1}(b, i) = \begin{cases} \min(\frac{1}{\alpha}(\sqrt{\beta X(b)i} - \beta i), P_{max}) & i \leq \frac{X(b)}{\beta} \\ 0 & i > \frac{X(b)}{\beta}. \end{cases} \quad (7)$$

where  $X(b)$  is an increasing function of the backlog  $b$ ,  $\alpha > 1$  and  $\beta > 0$ , which guarantees that  $s^1(p, i) \leq 1$ . An entire family of PCMA-1 algorithms can be obtained by modifying parameters  $X(b)$ ,  $\alpha$  and  $\beta$ .

- PCMA-2: For transmission success probability  $s^2(p, i) = 1 - \exp^{-\delta(p/i)}$ , the optimal power policy is given by

$$p^{*2}(b, i) = \begin{cases} \min(-\frac{i}{\delta} \log(\frac{i}{\delta X(b)}), P_{max}) & i \leq \delta X(b) \\ 0 & i > \delta X(b) \end{cases} \quad (8)$$

where  $\delta > 0$  is a parameter for PCMA-2 family of algorithms.

#### IV. SIMULATION RESULTS: PERFORMANCE IN ASYMMETRIC INTERFERENCE-LIMITED NETWORKS

In this section, we present the simulation results for PCMA and FM algorithm in asymmetric interference-limited two-link network. The same-link gains  $g_{11}$  and  $g_{22}$  are set to 1, while the cross-link gains are set to  $g_{12} = 10$  and  $g_{21} = 0.1$  for the simulation of asymmetric interference scenario in Fig. 1(b). So, the channel conditions are poor for the first link but are quite good for the second link. The thermal noise is assumed to be  $\eta = 0.1$ ,  $P_{max}$  is  $10^4$  or  $40dB$ . FM algorithm and dynamic Programming approach of PCMA algorithm is simulated in Matlab. PCMA-1 and PCMA-2 algorithms use the following parameters– the function  $X(b)$  given by  $X(b) = b + 4$ , parameters  $\alpha = \beta = 1$ , and  $\delta = 1$ . The SIR target is set to 0.99 for FM algorithm. The length of simulations is set to 30,000 time steps to ensure the convergence of results.

Fig. 2 (a) and (b) shows the simulation results when a symmetric traffic load ( $\lambda_1 = \lambda_2$ ) arrives at both the links. As the average traffic load is increased, Fig. 2 (a) shows the average backlog of the two links for different algorithms, while Fig. 2(b) shows the the average power of the both links. Because of asymmetry in the interference-limited network, FM algorithm is unable to support an average load of even 0.1, and the queues start exploding for both the links. For PCMA algorithms, the queue of the weak link explodes at a lower traffic load than the queue of the strong link, and thus, the performance of the network is dominated by the weak link, which is expected. Figures 3-5 shows the simulation results for an asymmetric traffic load when the average traffic load for the strong link  $\lambda_2$  is fixed and the average traffic load  $\lambda_1$  for the weak link increases. FM algorithm is not shown in these curves because queues start exploding for both links for traffic loads below 0.1. Figure 3 shows that for a low traffic load  $\lambda_2 = 0.2$  at the strong link, PCMA-2 algorithm allows the weak link to support traffic loads up to  $\lambda_1 = 0.7$  using PCMA-1 algorithm before the queue and the power start exploding. However, as the traffic load  $\lambda_2$  is increased to 0.4 and 0.6 for the strong link in Figures 4 and 5 respectively, the weak link supports much lower traffic loads. For example, for  $\lambda_2 = 0.6$  in Figure 5, the weak link supports a traffic load up to 0.3 and 0.5 respectively using PCMA-1 and PCMA-2 algorithm. This confirms that the performance of PCMA algorithms deteriorates when the average traffic load of the strong link is high, while the interference observed by the strong link is low.

#### V. PROPOSED VARIATION TO IMPROVE THE PERFORMANCE IN ASYMMETRIC INTERFERENCE-LIMITED NETWORK

In this section, based on the intuition gained from the simulation results above and a centralized algorithm [7], we introduce the concept of a penalty cost  $C(p)$  for strong link 2 as an increasing function of the interference created by the strong link and an indicator of the distress state of the weak link 1. For example, the penalty cost may be defined as  $C(p) = k g_{12} p / g_{11}$ ,

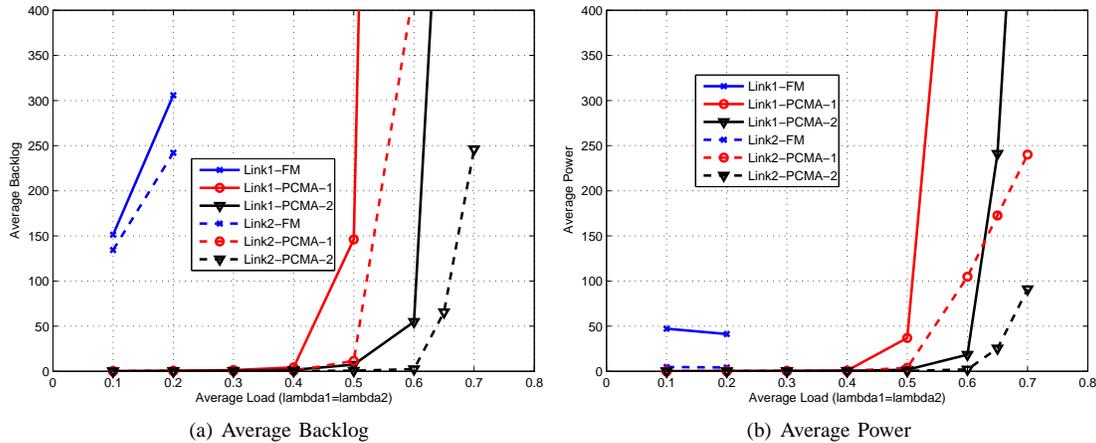


Fig. 2. Symmetric traffic load for both the links, i.e.  $\lambda_1 = \lambda_2$  ( $g_{11} = g_{22} = 1, g_{12} = 10, g_{21} = 0.1$ ).

where  $k$  is chosen to be high if weak link 1 indicates that its buffer is  $b_1$  exploding and otherwise  $k = 0$ . Accounting for this penalty cost  $C(p)$ , the DP recursion for the strong link in PCMA can be rewritten as –

$$V(b, i) = \inf_{p \geq 0} \{p + C(p) + B(b) + s(p, i) \sum_{j \in \mathcal{I}} q_{ij} V(b-1, j) + (1-s(p, i)) \sum_{j \in \mathcal{I}} q_{ij} V(b-1, j)\}, \quad (9)$$

which can be used to derive similar results as in the case of PCMA-1 and PCMA-2. Using the idea of penalty cost, modified PCMA algorithm may again exhibit TDMA type of behavior, however this is still being investigated through simulations and the appropriate  $C(p)$  still needs to be determined. Note that the idea of penalty cost introduces the concept that local network topology can be estimated by each link, even though the entire network topology may be unknown. This inherently introduces a notion of availability of partial network-state information, such as, information about distressed nodes in the network. Thus, even though there is no coordination between nodes in real-time and no central controller exists, the availability of partial network-state information helps the stronger links to make “informed” choices. One must note here that the stronger link must have some incentive to be “polite” to its neighboring weaker links, otherwise it will use an aggressive power policy to achieve its best performance.

## VI. CONCLUSIONS & FUTURE WORK

In this project report, we have investigated two-link interference-limited network with asymmetric interference through simulations of FM and PCMA algorithms. Preliminary simulations show that asymmetric interference lowers the performance of the network in terms of the average load it can support for both FM and PCMA algorithms, and the network performance is dominated by the weak link’s performance. As a part of future work, the modifications proposed in Section IV need to be evaluated to see if they improve the performance of PCMA in asymmetric interference scenarios. Furthermore, the algorithms discussed in this paper assume that the transition probabilities for interference states is known for the links, while learning them through reinforcement learning would be more valuable for practical networks. One approach for reinforcement learning is proposed in [8], however, the proposed approach is rather complex and other simpler approaches merit additional study especially in the asymmetric interference scenarios.

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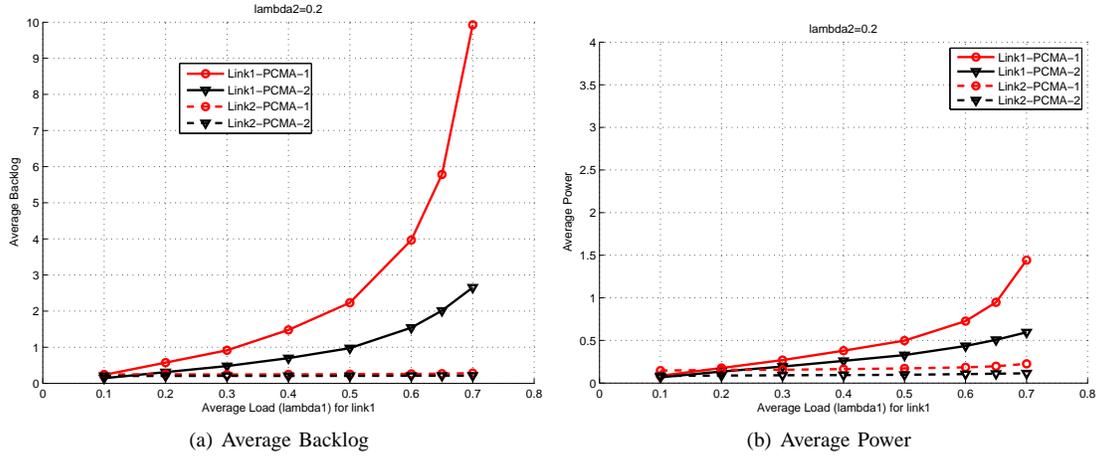


Fig. 3. Asymmetric traffic load for the two links.  $\lambda_2 = 0.2$  is fixed for the strong link while  $\lambda_1$  increases for the weak link. ( $g_{11} = g_{22} = 1$ ,  $g_{12} = 10, g_{21} = 0.1$ ).

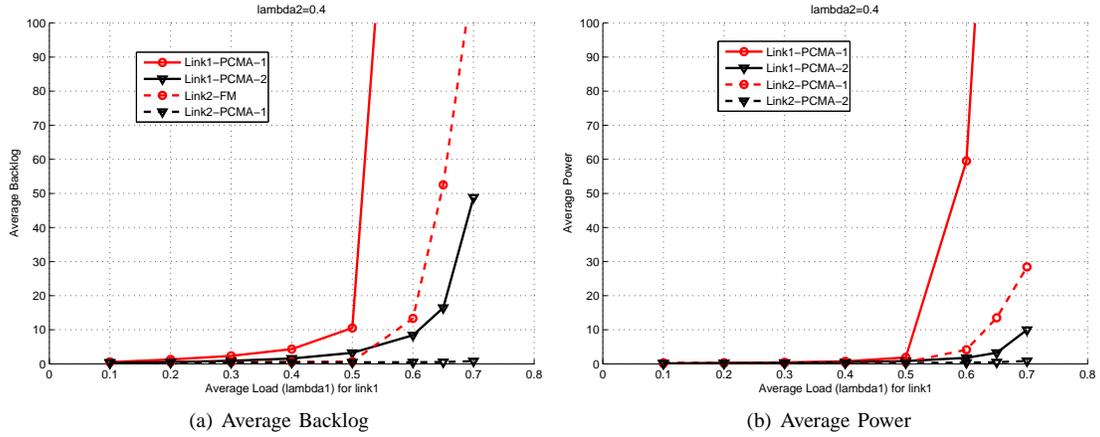


Fig. 4. Asymmetric traffic load for the two links.  $\lambda_2 = 0.4$  is fixed for the strong link while  $\lambda_1$  increases for the weak link. ( $g_{11} = g_{22} = 1$ ,  $g_{12} = 10, g_{21} = 0.1$ ).

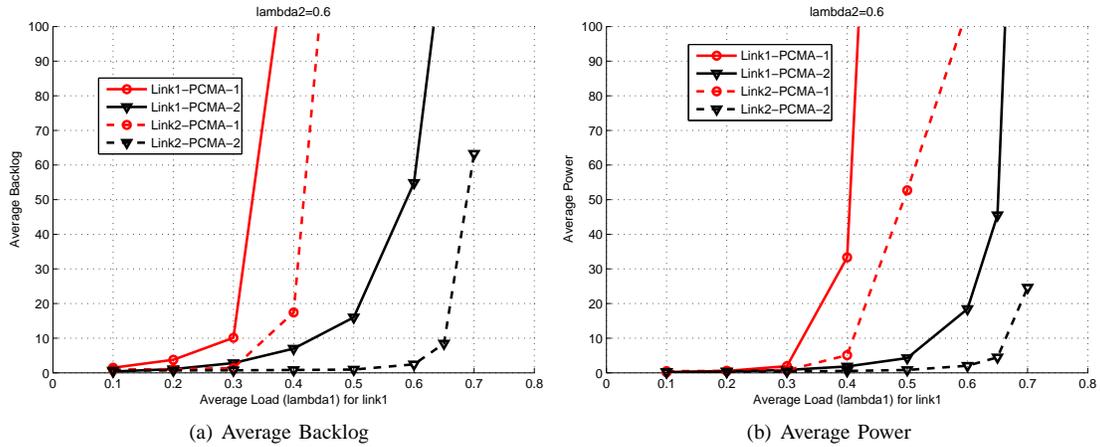


Fig. 5. Asymmetric traffic load for the two links.  $\lambda_2 = 0.6$  is fixed for the strong link while  $\lambda_1$  increases for the weak link. ( $g_{11} = g_{22} = 1$ ,  $g_{12} = 10, g_{21} = 0.1$ ).