

Classification of Fast Magnetic Resonance Image Reconstruction Using Matching Pursuit Family Algorithm

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1 Introduction

Magnetic Resonance Imaging (MRI) is one of imaging machine that has a low throughput outcome. A single patient can take approximately 30 minutes to an hour of measurement depending on the part of the body being measured. The lengthy process bottlenecked the income generated by a single machine, and increasing discomforts for patients as well.

There is a huge motivation to increase the throughput of MRI machine output by cutting down the number of image sample below Nyquist threshold. Hence dramatically decreasing the measurement time and increasing the number of patients that can use the MRI machine. Unfortunately, the reduction of number of image sampling will increase the difficulty and cost in image reconstruction process considerably. For example, a brute force image reconstruction could take weeks to reconstruct and image with 10% sampling.

Compressed Sensing (CS) is a recently introduced theory for the recovery of sparse signals from limited amount of measurement data. There are many novel solutions to solve such reconstruction. This paper will discuss the Matching Pursuit family technique such as Matching Pursuit, Orthogonal Matching Pursuit, Regularized Orthogonal Matching Pursuit, Compressive Sampling Matching Pursuit and Subspace Pursuit in classifying the image. *“Matching Pursuit family is a general, greedy, sparse function approximation scheme with the squared error loss, which iteratively adds new functions (i.e. basis functions) to the linear expansion. If we take as “dictionary of functions” the functions $d_i(\cdot)$ of the form $K(\cdot, x_i)$ where x_i is the input part of training example, then the linear expansion has essentially the same form as Support Vector Machine”*[1]. Our goal is to use Matching Pursuit algorithm to determine which parts of the pixel contains an image and which parts do not, while maintaining image quality and time cost. We finally discuss the optimum algorithm for solving the problem.

2 Problem Formulation

In MRI, the measurement can be represented by a linear combination of vector sets in Φ such that:

$$f = \Phi\alpha$$

Where f is the measured Fourier transformed signal in the k-space domain. Φ is the dictionary and α is the target sparse reconstructed image, such that:

$$f \in \mathbb{R}^n, \Phi \in \mathbb{R}^{n \times m}, \alpha \in \mathbb{R}^m$$

In CS, the problem arises as we decrease the number of measurement data significantly, such that $n > m$. Therefore the dimensionality of full row rank Φ becomes an under-deterministic problem with multiple solutions. Moreover, the solution estimation difficulty is increasing exponentially as the dimension scale is increased. The problem is further increased by inefficient implementation of Matching Pursuit family algorithm to solve larger dimension scale,

such as an image. Fortunately, if the signal α is sparse with k non-zero feature or element, the solution may be unique.

3 Matching Pursuit Family Algorithm

There are several Matching Pursuit family algorithms that will be studied here: Matching Pursuit (MP) [2], Orthogonal Matching Pursuit (OMP) [3], Regularized Orthogonal Matching Pursuit (ROMP) [4], Compressive Sampling Matching Pursuit (CoSaMP) [5] and Subspace Pursuit (SP) [6]. Only MP algorithm is shown.

3.1 Matching Pursuit

Mallat and Zhang proposed a scheme to recover a sparse signal by using a method called Matching Pursuit.

$$f = \hat{f}_k + r_k$$

Where \hat{f}_k is the approximation of f after k -th iteration, and r_k is the residual and is orthogonal to \hat{f}_k . Let α_k be the coefficients of the k chosen vectors from normalized Φ . Let ϕ_k be the k -th chosen vector, the algorithm will choose α_k with the largest magnitude element from the proxy with the following:

$$\|r_{k+1}\|^2 = \|r_k\|^2 - |\langle r_k, \phi_{k+1} \rangle|^2$$

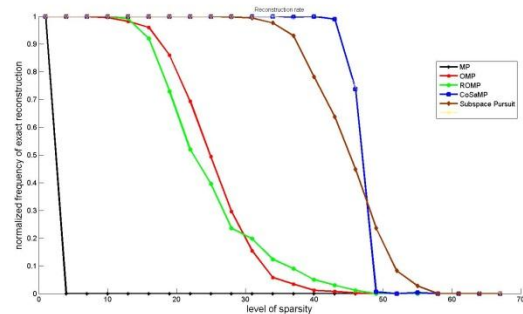
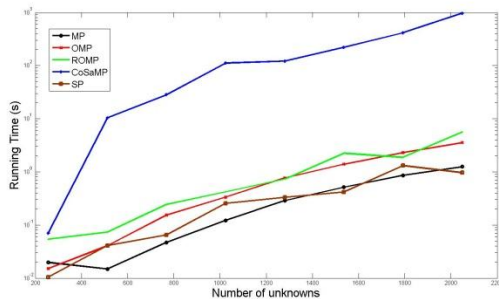
We want to minimize the next residual by maximizing the proxy $\langle r_k, \phi_{k+1} \rangle$.

Algorithm 1 MP:

1. Normalized Φ
2. Initialized $r_0 = f$
3. Loop until k :
 - a. Choose unpicked index of the largest proxy
 - b. $\alpha_i =$ de-normalized correlation value
 - c. $r_{i+1} = r_i - \phi_{i+1} \cdot \alpha_{i+1}$

4 Inverse Problem

Given a dictionary Φ and a measurement f , we want to reconstruct the sparse signal α , such that it contains only k number of non-zero elements. The purpose of this experiment is to compare the accuracy of the reconstructed signal to the true solution and the run time of each algorithms to finish the computation. Furthermore, the behavior of the run time will be used to predict the feasibility of the algorithms in large scale problem.



Algorithms	Current Implementation	Theoretical
MP	$O(mnk)$	$O(mnk)$
OMP	$O(mnk)$	$O(mnk)$
ROMP	$O(mk^3)$	$O(mnk)$
CoSaMP	$O(mk^3)$	$O(mnk)$
SP	$O(mk^3)$	$O(mnk)$

The current algorithms implementation is still inefficient for solving the inverse problem. As shown in the figure, as the number of dimension is increased, the running time will be increased as well. The bottleneck of the implementation occurs at the computation of the projection: $\alpha = (\Phi^* \Phi)^{-1} \Phi^* f$. At this point ROMP algorithm will be dropped from comparison study.

In the next section, we will discuss a novel way to solve the bottleneck problem using fixed point iteration called Richardson Iteration, which arguably faster than Newton's Method, to achieve the theoretical computation cost.

5 Richardson Iteration

Suppose we want to solve:

$$\min \|f - \Phi \alpha\|$$

let Φ be an orthonormal tall matrix and let the linear equation be:

$$(\Phi^* \Phi) \alpha = \Phi^* f$$

$$\Phi^* \Phi = I + M$$

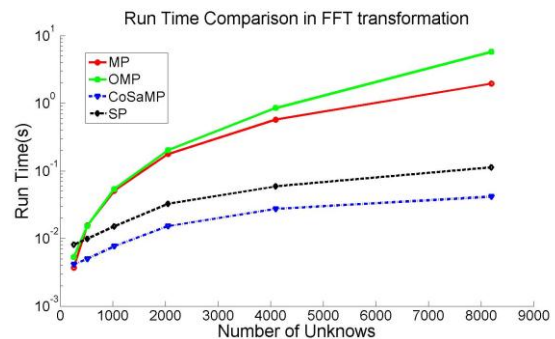
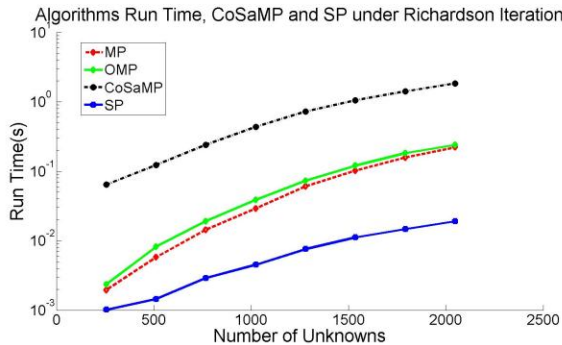
Joining the above equations yields:

$$\alpha_{i+1} = \Phi^* f - M \alpha_i$$

Algorithm 2 Richardson Iteration:

1. Calculate proxy = $\Phi^* f$
2. Initialized $\alpha_0 = 0$
3. Form $M = \Phi^* \Phi - I$
4. Repeat
 - $\alpha_{i+1} = \Phi^* f - M \alpha_i$
5. Until $\|\alpha_{i+1} - \alpha_i\| < \varepsilon$

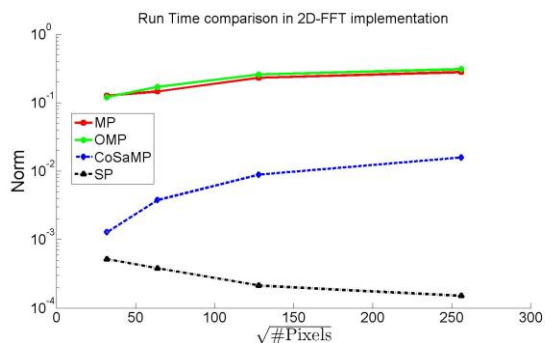
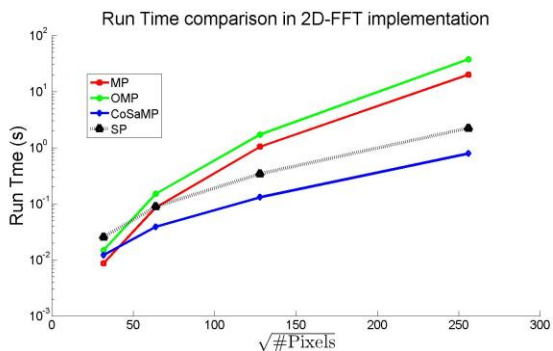
5.1 Richardson Iteration Implementation on CoSaMP and SP



Compared to previous figures in section 4, the new implementation of CoSaMP and SP with Richardson Iteration now scale well as the dimension of the problem becomes large. CoSaMP and SP performances are also better than MP and OMP performances.

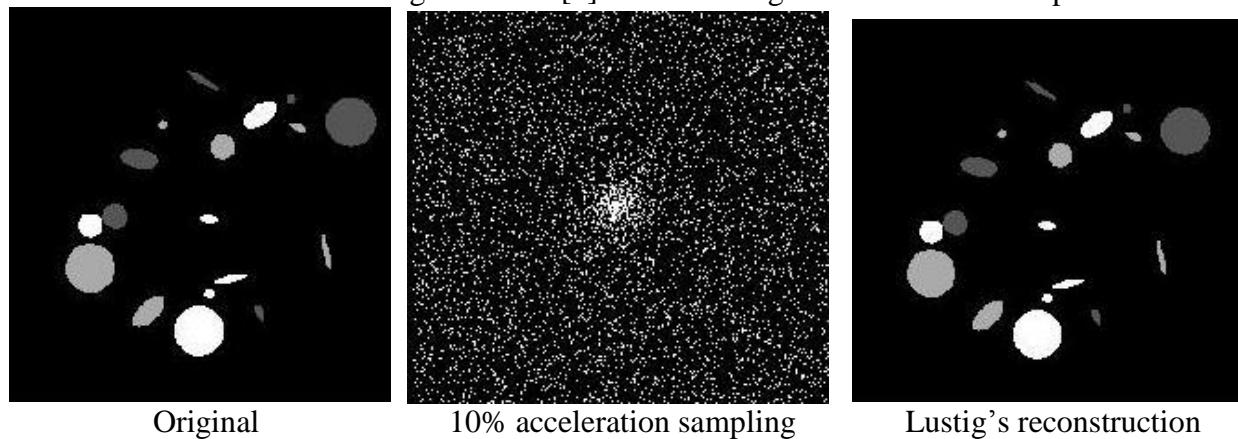
6 2D-FFT Implementation and Large Scale Problem

At this point, we will further increase the magnitude of dimension to 256 by 256 pixels to mimic the original MRI image dimension. At this point, we will drop MP and OMP from discussion as it is shown that CoSaMP and SP performed better compared to MP and OMP as the dimension is increased.

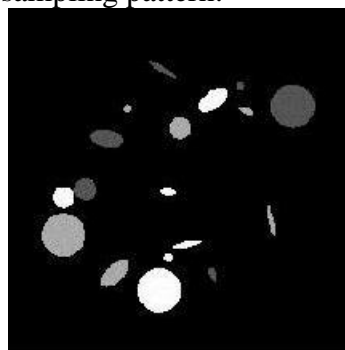


7 Image Reconstruction Comparison

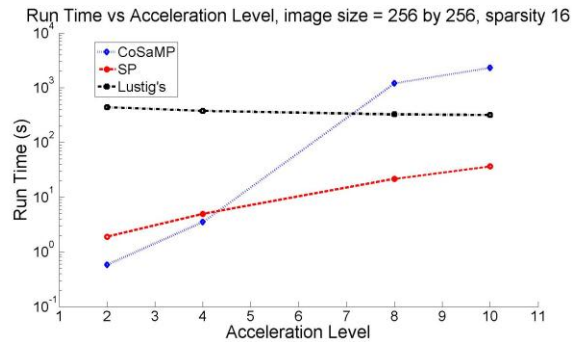
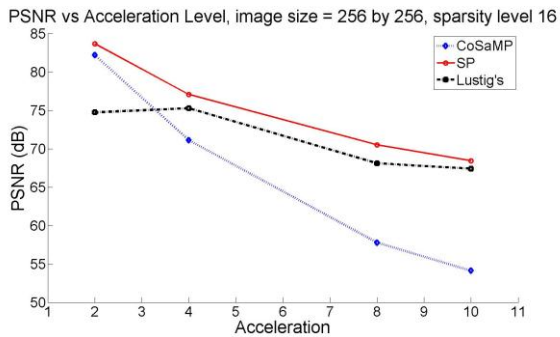
We will introduce Lustig's scheme [7] for MRI image reconstruction comparison.



With the same sampling pattern:



Qualitatively, all 3 reconstructed images are comparable. We will provide quantitative comparison based on the Peak Signal to Noise Ratio (PSNR) to compare reconstruction quality and computation run time to compare the cost.



As the number of acceleration is increased, the reconstructed quality of CoSaMP is decreasing much more than SP. It is also more costly in the experiments compared to SP. While SP are generally perform better than Lustig's scheme and CoSaMP.

7 Conclusion

For overall, CoSaMP does not provide better reconstruction compared to the results from SP. Under certain conditions, SP generally provide us with better run time and accuracy compared to the algorithms implemented by Lustig and CoSaMP. Therefore, SP is the best choice among the studied algorithms in this project.

8 References

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